We consider a stochastic decentralized decision making problem in which there is one leader and three followers. Assume the control vector is $\boldsymbol{x} = (x_1, x_2)$, and the control vectors of the three followers are $\boldsymbol{y}_i = (y_{i1}, y_{i2})$, $i = 1, 2, 3$. Let us solve the following expected value multilevel programming,

$$
\begin{cases}
\max\limits_{x_1,x_2} E\left[\sqrt{x_1x_2 + y_{11}^*y_{12}^* + y_{21}^*y_{22}^* + y_{31}^*y_{32}^* + \xi^2}\right] \\
\text{subject to:} \\
\quad x_1 + x_2 \le 10,\ x_1 \ge 0,\ x_2 \ge 0 \\
\quad (y_{11}^*, y_{21}^*, y_{31}^*, y_{12}^*, y_{22}^*, y_{32}^*) \text{ solves the problems} \\
\quad \begin{cases}
\max\limits_{y_{11},y_{12}} E\left[\sqrt{x_1^2 + y_{11}^2 + 2y_{12}^2 + \xi_1^2}\right] \\
\text{subject to:} \\
\quad y_{11} + 2y_{12} \le x_2,\ y_{11} \ge 0,\ y_{12} \ge 0
\end{cases} \\
\quad \begin{cases}
\max\limits_{y_{21},y_{22}} E\left[\sqrt{x_1^2 + y_{21}^2 + 2y_{22}^2 + \xi_2^2}\right] \\
\text{subject to:} \\
\quad 2y_{21} + 3y_{22} \le x_2,\ y_{21} \ge 0,\ y_{22} \ge 0
\end{cases} \\
\quad \begin{cases}
\max\limits_{y_{31},y_{32}} E\left[\sqrt{x_1^2 + y_{31}^2 + 2y_{32}^2 + \xi_3^2}\right] \\
\text{subject to:} \\
\quad 3y_{31} + 4y_{32} \le x_2,\ y_{31} \ge 0,\ y_{32} \ge 0
\end{cases}
\end{cases}
$$

where ξ_1, ξ_2, ξ_3, ξ are normally distributed random variables $\mathcal{N}(1,1)$, $\mathcal{N}(2,1)$, $\mathcal{N}(3,1)$, $\mathcal{N}(4,1)$, respectively.

In order to solve this problem, we generate input-output data for the uncertain functions

$$
\begin{aligned}
U_0 : (\boldsymbol{x}, \boldsymbol{y}_1, \boldsymbol{y}_2, \boldsymbol{y}_3) &\to E\left[\sqrt{x_1x_2 + y_{11}y_{12} + y_{21}y_{22} + y_{31}y_{32} + \xi^2}\right], \\
U_1 : (\boldsymbol{x}, \boldsymbol{y}_1) &\to E\left[\sqrt{x_1^2 + y_{11}^2 + 2y_{12}^2 + \xi_1^2}\right], \\
U_2 : (\boldsymbol{x}, \boldsymbol{y}_2) &\to E\left[\sqrt{x_1^2 + y_{21}^2 + 2y_{22}^2 + \xi_2^2}\right], \\
U_3 : (\boldsymbol{x}, \boldsymbol{y}_3) &\to E\left[\sqrt{x_1^2 + y_{31}^2 + 2y_{32}^2 + \xi_3^2}\right]
\end{aligned}
$$

by stochastic simulation. Then we train an NN to approximate the uncertain functions U_i, $i = 0, 1, 2, 3$.

A run of the hybrid intelligent algorithm shows that the stackelberg-Nash equilibrium is

$$
\begin{aligned}
&(x_1^*, x_2^*) = (4.670, 5.326), \\
&(y_{11}^*, y_{12}^*) = (5.326, 0.000), \\
&(y_{21}^*, y_{22}^*) = (2.663, 0.000), \\
&(y_{31}^*, y_{32}^*) = (0.000, 1.332)
\end{aligned}
$$

and the optimal objective values of the leader and three followers are 6.477, 7.218, 5.811, 5.937, respectively.