We consider a stochastic decentralized decision making problem in which there is one leader and three followers. Assume the control vector is \( x = (x_1, x_2) \), and the control vectors of the three followers are \( y_i = (y_{i1}, y_{i2}) \), \( i = 1, 2, 3 \). Let us solve the following expected value multilevel programming,

\[
\begin{align*}
\max_{x_1, x_2} \ E \left[ \sqrt{x_1 x_2 + y_{11}^* y_{12}^* + y_{21}^* y_{22}^* + y_{31}^* y_{32}^* + \xi^2} \right] \\
\text{subject to:} \\
x_1 + x_2 \leq 10, \ x_1 \geq 0, \ x_2 \geq 0 \\
(y_{11}, y_{21}, y_{31}, y_{12}, y_{22}, y_{32}) \text{ solves the problems} \\
\begin{align*}
\max_{y_{11}, y_{12}} & \ E \left[ \sqrt{x_1^2 + y_{11}^2 + 2y_{12}^2 + \xi_1^2} \right] \\
\text{subject to:} & \\
y_{11} + 2y_{12} \leq x_2, \ y_{11} \geq 0, \ y_{12} \geq 0 \\
\max_{y_{21}, y_{22}} & \ E \left[ \sqrt{x_1^2 + y_{21}^2 + 2y_{22}^2 + \xi_2^2} \right] \\
\text{subject to:} & \\
2y_{21} + 3y_{22} \leq x_2, \ y_{21} \geq 0, \ y_{22} \geq 0 \\
\max_{y_{31}, y_{32}} & \ E \left[ \sqrt{x_1^2 + y_{31}^2 + 2y_{32}^2 + \xi_3^2} \right] \\
\text{subject to:} & \\
3y_{31} + 4y_{32} \leq x_2, \ y_{31} \geq 0, \ y_{32} \geq 0 
\end{align*}
\end{align*}
\]

where \( \xi_1, \xi_2, \xi_3, \xi \) are normally distributed random variables \( \mathcal{N}(1, 1), \mathcal{N}(2, 1), \mathcal{N}(3, 1), \mathcal{N}(4, 1) \), respectively.

In order to solve this problem, we generate input-output data for the uncertain functions

\[
\begin{align*}
U_0 : (x, y_1, y_2, y_3) & \rightarrow E \left[ \sqrt{x_1 x_2 + y_{11} y_{12} + y_{21} y_{22} + y_{31} y_{32} + \xi^2} \right], \\
U_1 : (x, y_1) & \rightarrow E \left[ \sqrt{x_1^2 + y_{11}^2 + 2y_{12}^2 + \xi_1^2} \right], \\
U_2 : (x, y_2) & \rightarrow E \left[ \sqrt{x_1^2 + y_{21}^2 + 2y_{22}^2 + \xi_2^2} \right], \\
U_3 : (x, y_3) & \rightarrow E \left[ \sqrt{x_1^2 + y_{31}^2 + 2y_{32}^2 + \xi_3^2} \right]
\end{align*}
\]

by stochastic simulation. Then we train an NN to approximate the uncertain functions \( U_i \), \( i = 0, 1, 2, 3 \).

A run of the hybrid intelligent algorithm shows that the Stackelberg-Nash equilibrium is

\[
(x_1^*, x_2^*) = (4.670, 5.326), \\
(y_{11}^*, y_{12}^*) = (5.326, 0.000), \\
(y_{21}^*, y_{22}^*) = (2.663, 0.000), \\
(y_{31}^*, y_{32}^*) = (0.000, 1.332)
\]

and the optimal objective values of the leader and three followers are 6.477, 7.218, 5.811, 5.937, respectively.