

Now we reconsider the above stochastic decentralized decision making problem. Suppose that the leader and followers want to optimize their optimistic return with a given confidence level subject to some chance constraints, then the stochastic decentralized decision making problem can be formulated as the following chance-constrained multilevel programming,

$$\left\{ \begin{array}{l} \max_{x_1, x_2} \bar{F} \\ \text{subject to:} \\ \Pr \left\{ \sqrt{x_1 x_2 + y_{11}^* y_{12}^* + y_{21}^* y_{22}^* + y_{31}^* y_{32}^* + \xi^2} \geq \bar{F} \right\} \geq 0.9 \\ x_1 + x_2 \leq 10, x_1 \geq 0, x_2 \geq 0 \\ (y_{11}^*, y_{21}^*, y_{31}^*, y_{12}^*, y_{22}^*, y_{32}^*) \text{ solves the problems} \\ \left\{ \begin{array}{l} \max_{y_{11}, y_{12}} \bar{f}_1 \\ \text{subject to:} \\ \Pr \left\{ \sqrt{x_1^2 + y_{11}^2 + 2y_{12}^2 + \xi_1^2} \geq \bar{f}_1 \right\} \geq 0.9 \\ y_{11} + 2y_{12} \leq x_2, y_{11} \geq 0, y_{12} \geq 0 \end{array} \right. \\ \left\{ \begin{array}{l} \max_{y_{21}, y_{22}} \bar{f}_2 \\ \text{subject to:} \\ \Pr \left\{ \sqrt{x_1^2 + y_{21}^2 + 2y_{22}^2 + \xi_2^2} \geq \bar{f}_2 \right\} \geq 0.9 \\ 2y_{21} + 3y_{22} \leq x_2, y_{21} \geq 0, y_{22} \geq 0 \end{array} \right. \\ \left\{ \begin{array}{l} \max_{y_{31}, y_{32}} \bar{f}_3 \\ \text{subject to:} \\ \Pr \left\{ \sqrt{x_1^2 + y_{31}^2 + 2y_{32}^2 + \xi_3^2} \geq \bar{f}_3 \right\} \geq 0.9 \\ 3y_{31} + 4y_{32} \leq x_2, y_{31} \geq 0, y_{32} \geq 0 \end{array} \right. \end{array} \right.$$

where ξ_1, ξ_2, ξ_3, ξ are normally distributed random variables $\mathcal{N}(1, 1)$, $\mathcal{N}(2, 1)$, $\mathcal{N}(3, 1)$, $\mathcal{N}(4, 1)$, respectively.

In order to solve this problem, we generate input-output data for the uncertain functions

$$\begin{aligned} U_0 : (\mathbf{x}, \mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3) &\rightarrow \max \left\{ \bar{F} \mid \Pr \left\{ \sqrt{x_1 x_2 + y_{11}^* y_{12}^* + y_{21}^* y_{22}^* + y_{31}^* y_{32}^* + \xi^2} \geq \bar{F} \right\} \geq 0.9 \right\}, \\ U_1 : (\mathbf{x}, \mathbf{y}_1) &\rightarrow \max \left\{ \bar{f}_1 \mid \Pr \left\{ \sqrt{x_1^2 + y_{11}^2 + 2y_{12}^2 + \xi_1^2} \geq \bar{f}_1 \right\} \geq 0.9 \right\}, \\ U_2 : (\mathbf{x}, \mathbf{y}_2) &\rightarrow \max \left\{ \bar{f}_2 \mid \Pr \left\{ \sqrt{x_1^2 + y_{21}^2 + 2y_{22}^2 + \xi_2^2} \geq \bar{f}_2 \right\} \geq 0.9 \right\}, \\ U_3 : (\mathbf{x}, \mathbf{y}_3) &\rightarrow \max \left\{ \bar{f}_3 \mid \Pr \left\{ \sqrt{x_1^2 + y_{31}^2 + 2y_{32}^2 + \xi_3^2} \geq \bar{f}_3 \right\} \geq 0.9 \right\} \end{aligned}$$

by stochastic simulation. Then we train an NN to approximate the uncertain functions U_i , $i = 0, 1, 2, 3$.

A run of the hybrid intelligent algorithm shows that the stackelberg-Nash equilibrium is

$$\begin{aligned} (x_1^*, x_2^*) &= (4.698, 5.301), \\ (y_{11}^*, y_{12}^*) &= (5.301, 0.000), \\ (y_{21}^*, y_{22}^*) &= (2.650, 0.000), \\ (y_{31}^*, y_{32}^*) &= (0.000, 1.325), \end{aligned}$$

the optimal objective values of the leader and three followers are 5.567, 7.086, 5.444, 5.352, respectively.