A Chance-Constrained Programming Model for Inverse Spanning Tree Problem with Uncertain Edge Weights

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Abstract

The inverse spanning tree problem is to make a given spanning tree be a minimum spanning tree on a connected graph via a minimum perturbation on its edge weights. In this paper, a chance-constrained programming model is proposed to handle the inverse spanning tree problem where the edge weights are assumed to be uncertain variables. It is shown that such an uncertain minimum spanning tree can be characterized by some constraints on the paths of the graph. Consequently, the proposed model can be reformulated into a deterministic programming model. Furthermore, when the edge weights are linear uncertain variables, the corresponding model reduces to a linear programming problem and can be solved efficiently.

Keywords: Minimum Spanning Tree, Inverse Optimization, Uncertain Programming

1. Introduction

The inverse spanning tree problem or inverse minimum spanning tree problem is a type of inverse optimization problems. For a connected graph with edge weights, the inverse spanning tree problem is to modify the weights as little as possible so that a given spanning tree becomes a minimum spanning tree of the graph with respect to the new weights, which means the deviation incurred by the modification is to be minimized.

The inverse spanning tree problem was first studied by Zhang et al. [13]. Since that, much work has been done on the inverse spanning tree problem because many practical problems can be handled in this framework (see, e.g., [3,4,12]). For the classical inverse spanning tree problem and some of its derivatives, some efficient algorithms are available (see, e.g., [2,5,6,11,17]). However, in some practical circumstances, the edge weights cannot be explicitly determined. Although an approximation by random or fuzzy variables may be applied, see, e.g., [14,15], it does not work when there are not enough data for estimation or when the edge weights possess an unobservable nature. In order to deal with this situation, the notion of an uncertain spanning tree has been proposed in [10,18] where the edge weights are characterized by uncertain variables in the sense of Liu [7]. Furthermore, Zhang et al. [16] provided the notation of an uncertain $\alpha$-minimum spanning tree using the chance constraint. In this paper, the inverse minimum spanning tree problem with uncertain edge weights is considered, which is referred to as the uncertain inverse spanning tree problem for convenience. Analogous to the path optimality condition identified by Ahuja et al. [1], a generalization is developed to characterize the uncertain $\alpha$-minimum spanning tree in the uncertain environment. Consequently, the uncertain inverse spanning tree problem can be reformulated into a deterministic programming model. Furthermore, when the edge weights are linear uncertain variables, the corresponding model reduces to a linear programming problem and can be solved efficiently taking the advantage of some well developed software packages.

The rest of this paper is organized as follows. Section 2 briefly reviews the classic inverse spanning tree problem as well as some basic notions and results in uncertainty theory. The notion of the uncertain $\alpha$-minimum spanning tree is recalled in Section 3 and further investigated. In Section 4, a chance-constrained programming model is formulated for the uncertain inverse spanning tree problem and transformed into its equivalent deterministic counterpart. Besides, some illustrative examples are presented in Section 5.

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2. Preliminaries

In this section, the classic inverse spanning tree problem is briefly reviewed. Besides, some basic notions and results of uncertainty theory are recalled which are indispensable to handle the uncertain inverse spanning tree problem.

2.1. Classic inverse spanning tree problem

Let $G = (V, E)$ denote a connected graph consisting of the vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and the edge set $E = \{1, 2, \ldots, m\}$. A spanning tree $T = T(V, S)$ of $G$ is a connected acyclic subgraph containing all vertices. For simplicity, we denote a spanning tree $T$ by its edge set $S$ throughout this paper.

**Definition 1 (Minimum Spanning Tree)** Given a connected graph $G = (V, E)$ with edge weights $x_i, i \in E = \{1, 2, \ldots, m\}$, a spanning tree $T^0$ is said to be a minimum spanning tree if

\[
\sum_{i \in T^0} x_i \leq \sum_{j \in T} x_j
\]

holds for any spanning tree $T$.

The classic inverse spanning tree problem is to find some new edge weights such that a given spanning tree $T^0$ is a minimum spanning tree with respect to the new edge weights and the total change of the edge weights is minimized.

As an illustration, a graph with 6 vertices and 10 edges is shown in Figure 1, where $c_i$ and $x_i$ denote the original and new weights on edge $i$, respectively. The edges (solid lines) define a spanning tree $T^0$ and hence called tree edges while the rest edges (dash lines), that are not in $T^0$, are called non-tree edges. It is well-known that a spanning tree induces a unique path between every pair of vertices. Especially, for any non-tree edge $j$, there must be a unique path between the vertices of edge $j$ containing only the tree edges. This path is called the tree path of edge $j$, and is denoted by $P_j$. For instance, the tree path of non-tree edge $BD$ in Figure 1 is $AB-AE-DE$, i.e., $P_0 = \{1, 3, 5\}$. 

![Figure 1. An example of inverse spanning tree problem](image)

In order to formulate the inverse spanning tree problem, an alternative representation of the minimum spanning tree is needed, which is referred to as the path optimality condition. The path optimality condition characterizes the minimum spanning tree by a set of constraints on non-tree edges and their tree paths.

**Lemma 1 (Ahuja et al. [1], Path Optimality Condition)** Given a connected graph $G = (V, E)$ with edge weights $x_i, i \in E = \{1, 2, \ldots, m\}$, a spanning tree $T^0$ is a minimum spanning tree if and only if

\[
x_i - x_j \leq 0, \quad j \in E \setminus T^0, \ i \in P_j
\]

where $E \setminus T^0$ is the set of non-tree edges and $P_j$ is the corresponding tree path of edge $j$.

According to Lemma 1, the classic inverse spanning tree problem can be formulated as
\[
\begin{align*}
\min_{x} & \sum_{i=1}^{m} |x_i - c_i| \\
\text{subject to:} & \quad x_i - x_j \leq 0, \ j \in E \setminus T^0, i \in P_j
\end{align*}
\]

where \(c_i\) and \(x_i\) are the original and new weights of edge \(i, i \in E\), respectively. Some efficient algorithms are available to solve this model (see, e.g., [2,5,11]). Note that in [11] other types of objective functions are considered as well for the inverse spanning tree problems.

2.2. Uncertain variables

Uncertainty theory, initiated by Liu [7], provides a framework to deal with indeterminacy phenomena when there is a lack of observed data. In uncertainty theory, an uncertain variable \(\xi\) is defined as a measurable function from an uncertainty space \((\mathcal{U}, \mathcal{L}, \mathcal{M})\) to the set of real numbers, where \(\mathcal{U}\) is a nonempty set, \(\mathcal{L}\) is a \(\sigma\)-algebra over \(\mathcal{U}\), and \(\mathcal{M}\) is an uncertain measure. An uncertain measure \(\mathcal{M}\) is a set function satisfying the conditions of normality, self-duality, and countable subadditivity. The reader may refer to [9] for more details.

**Definition 2** (Liu [7]) The uncertainty distribution \(\Phi\) of an uncertain variable \(\xi\) is defined by

\[
\Phi(x) = \mathcal{M}\{\xi \leq x\}
\]

for any real number \(x\).

For example, a linear uncertain variable \(\xi\) has an uncertainty distribution

\[
\Phi(x) = \begin{cases} 0, & \text{if } x \leq a \\ (x - a)/(b - a), & \text{if } a < x \leq b \\ 1, & \text{if } x > b
\end{cases}
\]

denoted by \(\xi \sim \mathcal{L}(a,b)\), where \(a\) and \(b\) are real numbers with \(a < b\).

An uncertainty distribution \(\Phi\) is said to be regular if its inverse function \(\Phi^{-1}(\cdot)\) exists and is unique for each \(\xi \in (0,1)\). It is clear that a linear uncertainty distribution \(\mathcal{L}(a,b)\) is regular, and its inverse uncertainty distribution is

\[
\Phi^{-1}(a) = (1-a)a + ab.
\]

The inverse uncertainty distribution plays an important role in the arithmetic operations of independent uncertain variables.

**Definition 3** (Liu [8]) The uncertain variables \(\xi_1, \xi_2, \ldots, \xi_n\) are said to be independent if

\[
\mathcal{M}\left(\bigcap_{i=1}^{n} (\xi_i \in B_i)\right) = \mathcal{M}(\xi_1 \in B_1) \cap \ldots \cap \mathcal{M}(\xi_n \in B_n)
\]

for any Borel sets \(B_1, B_2, \ldots, B_n\) of real numbers.

**Lemma 2** (Liu [9]) Let \(\xi_1, \xi_2, \ldots, \xi_n\) be independent uncertain variables with regular uncertainty distributions \(\Phi_1, \Phi_2, \ldots, \Phi_n\), respectively. If the function \(f(x_1, x_2, \ldots, x_n)\) is strictly increasing with respect to \(x_1, x_2, \ldots, x_k\) and strictly decreasing with respect to \(x_{k+1}, x_{k+2}, \ldots, x_n\), then

\[
\xi = f(\xi_1, \xi_2, \ldots, \xi_n)
\]

is an uncertain variable with inverse uncertainty distribution

\[
\Phi^{-1}(\cdot) = f(\Phi_1^{-1}(\cdot), \ldots, \Phi_k^{-1}(\cdot), \Phi_{k+1}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)).
\]

3. Uncertain path optimality conditions

In order to deal with the minimum spanning tree when the edge weights are uncertain, Zhang et al. [16] proposed the notion of uncertain \(\alpha\)-minimum spanning tree.

**Definition 4** (Zhang et al. [16], Uncertain \(\alpha\)-Minimum Spanning Tree) Given a connected graph \(G = (V,E)\) with independent uncertain edge weights \(\xi_i, i \in E\), a spanning tree \(T^0\) is said to be an uncertain \(\alpha\)-minimum spanning tree if

\[
\mathcal{M}\left(\sum_{i \in T^0} \xi_i \leq \sum_{j \in T^0} \xi_j\right) \geq \alpha
\]
holds for any spanning tree $T$, where $\alpha$ is a predetermined confidence level.

Definition 4 implies that an uncertain $\alpha$-minimum spanning tree has a chance not less than $\alpha$ of not having an uncertain weight larger than every other spanning tree, which is intuitively reasonable.

In Section 2.1, an alternative characterization of the minimum spanning tree is presented in Lemma 1, which can be extended as well to the uncertain scenario.

**Theorem 1 (Uncertain Path Optimality Condition I)** Given a connected graph $G = (V, E)$ with independent uncertain weights $\xi_i, i \in E$, and a confidence level $\alpha$, a spanning tree $T^0$ is an uncertain $\alpha$-minimum spanning tree if and only if

$$M\{\xi_i - \xi_j \leq 0\} \geq \alpha, \quad j \in E \setminus T^0, i \in P_j$$

where $E \setminus T^0$ is the set of non-tree edges, and $P_j$ is the tree path of edge $j$.

**Proof:** Assume that $T^0$ is an uncertain $\alpha$-minimum spanning tree of $G$. For any $j \in E \setminus T^0$ and $i \in P_j$, a spanning tree $T'$ can be constructed by substituting edge $j$ for edge $i$ in $T^0$, i.e., $T' = T^0 \cup \{i\}$. It follows from Definition 4 that

$$M\left\{\sum_{i \in T^0} \xi_i - \sum_{j \in T'} \xi_j \right\} \geq \alpha.$$

By eliminating the common parts, we get

$$M\{\xi_i - \xi_j \leq 0\} \geq \alpha.$$

Conversely, suppose that $T^0$ satisfies the uncertain optimality condition (8). Let $T^\ast$ be an uncertain $\alpha$-minimum spanning tree and $T^0 \neq T^\ast$. For any edge $i$ satisfying $i \in T^0$ but $i \notin T^\ast$, the subgraph $T^\ast \cup \{i\}$ has a unique cycle. It follows from Definition 4 that

$$M\{\xi_i - \xi_j \leq 0\} \geq \alpha, \quad \forall j \in P_i.$$

Furthermore, since $T^0$ is a spanning tree, there is at least one edge $j \in T^\ast$ in the cycle satisfying $j \notin T^0$ and the cycle in the subgraph $T^0 \cup \{j\}$ contains edge $i$. Thus

$$M\{\xi_i - \xi_j \leq 0\} \geq \alpha$$

and hence $M\{\xi_i - \xi_j = 0\} \geq \alpha$. Consequently,

$$M\left\{\sum_{i \in T^0} \xi_i - \sum_{j \in T'} \xi_j \right\} \geq \alpha$$

which implies $T^0$ is also an uncertain $\alpha$-minimum spanning tree.

If the uncertain edge weights $\xi_i, i \in E$, are assumed to be independent with regular uncertainty distributions, by Lemma 2, an equivalent formulation of the path optimality condition can be obtained using the inverse uncertainty distributions.

**Lemma 3** Assume that $\xi_1, \xi_2, \ldots, \xi_m$ are independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_m$, respectively. Then the inequalities

$$M\{\xi_i - \xi_j \leq 0\} \geq \alpha, \quad j \in E \setminus T^0, i \in P_j$$

hold if and only if

$$\Phi_i^{-1}(\alpha) - \Phi_j^{-1}(1 - \alpha) \leq 0, \quad j \in E \setminus T^0, i \in P_j.$$  \hspace{1cm} (9)

**Proof:** Denote $f(\xi_i, \xi_j) = \xi_i - \xi_j$ for any given pair $(j, i)$ satisfying $j \in E \setminus T^0$ and $i \in P_j$. Since $f$ is a strictly increasing function with respect to $\xi_i$ and a strictly decreasing function with respect to $\xi_j$, it follows from Lemma 2 that the inverse uncertainty distribution of $f(\xi_i, \xi_j)$ is $\Phi_i^{-1}(\alpha) - \Phi_j^{-1}(1 - \alpha)$. Thus it is easy to verify that the constraint $M\{\xi_i - \xi_j \leq 0\} \geq \alpha$ is equivalent to

$$\Phi_i^{-1}(\alpha) - \Phi_j^{-1}(1 - \alpha) \leq 0.$$

**Theorem 2 (Uncertain Path Optimality Condition II)** Given a connected graph $G = (V, E)$ with independent uncertain weights $\xi_i, i \in E$, with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_m$, respectively, a spanning tree $T^0$ is an uncertain $\alpha$-minimum spanning tree if and only if

$$\Phi_i^{-1}(\alpha) - \Phi_j^{-1}(1 - \alpha) \leq 0, \quad j \in E \setminus T^0, i \in P_j$$

**Proof:** It is a direct consequence of Theorem 1 and Lemma 3.
4. Uncertain chance-constrained programming

In this section, the inverse spanning tree problem is discussed on a graph with uncertain edge weights. Let \( G = (V, E) \) be a connected graph with edge weights \( \xi_i, i \in E \). For each \( \xi_i \), its distribution is determined by a parameter \( c_i \), which can be modified accordingly. The uncertain inverse spanning tree problem is to find new parameters \( x_0, i \in E \), such that a given spanning tree becomes an uncertain \( \alpha \)-minimum spanning tree with respect to the new uncertain edge weights, and the modification \( \sum_{i=1}^{m} |x_i - c_i| \) is to be minimized.

Based on Theorem 1, the uncertain inverse spanning tree problem can be formulated as an uncertain chance-constrained programming model as

\[
\begin{align*}
\min_{x} \sum_{i=1}^{m} |x_i - c_i| \\
\text{subject to:} \\
M \{\xi_i(x_i) - \xi_j(x_j) \leq 0\} \geq \alpha, \quad j \in E \setminus T^0, i \in P_j
\end{align*}
\]  

(11)

where \( \alpha \) is a predetermined confidence level.

By the uncertain path optimality condition specified in Theorem 2, model (11) can be transformed to the following deterministic equivalent formulation,

\[
\begin{align*}
\min_{x} \sum_{i=1}^{m} |x_i - c_i| \\
\text{subject to:} \\
\Phi_i^{-1}(x_i, \alpha) - \Phi_j^{-1}(x_j, 1 - \alpha) \leq 0, \quad j \in E \setminus T^0, i \in P_j
\end{align*}
\]

(12)

where \( \Phi_i^{-1}, i \in E \), represent the inverse distributions of uncertain weights \( \xi_i \), with their distributions determined by the new parameters \( x_i, i \in E \).

Once the uncertainty distributions are specified, model (12) becomes a deterministic programming problem and may be solved by some well developed algorithms or software packages. Generally, it is a nonlinear programming problem and may require much computational effort. However, when the uncertain edge weights \( \xi_i, i \in E \), are linear uncertain variables, for example, \( \xi_i = L(a_i - x_i, b_i - x_i) \), it follows from (4) that the inverse uncertainty distribution of \( \xi_i \) is

\[
\Phi_i^{-1}(x_i, \alpha) = (1 - \alpha)(a_i - x_i) + \alpha(b_i - x_i) = a_i + \alpha(b_i - a_i) - x_i.
\]

(13)

Thus

\[
\Phi_i^{-1}(x_i, \alpha) - \Phi_j^{-1}(x_j, 1 - \alpha) = -x_i + x_j - K_{\xi}
\]

(14)

where \( K_{\xi} = b_i - a_i - \alpha(b_i - a_i) \) is a constant. Consequently, model (12) reduces to a linear programming problem

\[
\begin{align*}
\min_{x} \sum_{i=1}^{m} |x_i - c_i| \\
\text{subject to:} \\
x_j - x_i \leq K_{\xi}, \quad j \in E \setminus T^0, i \in P_j
\end{align*}
\]

(15)

Furthermore, by introducing auxiliary variables \( y_i^+, y_i^- \), \( i \in E \), where

\[
y_i^+ = \begin{cases} x_i - c_i, & \text{if } x_i \geq c_i \\ 0, & \text{if } x_i < c_i \end{cases}, \quad y_i^- = \begin{cases} c_i - x_i, & \text{if } x_i < c_i \\ 0, & \text{if } x_i \geq c_i \end{cases}
\]

(16)

the terms \( |x_i - c_i|, i \in E \), can be represented as

\[
|x_i - c_i| = y_i^+ + y_i^-,
\]

(17)

Taking (14) and (17) into model (15), it becomes

\[
\begin{align*}
\min_{x} \sum_{i=1}^{m} (y_i^+ + y_i^-) \\
\text{subject to:} \\
y_i^+ - y_i^- \leq K_{\xi}, \quad j \in E \setminus T^0, i \in P_j \\
y_i^+ \geq 0
\end{align*}
\]

(18)

which is a linear programming model and can be efficiently solved.
5. Computational examples

In order to illustrate the effectiveness of the model proposed above, in this section, a LAN reconstruction problem with 6 service centers and 10 bridges is considered (see Figure 2), where the solid lines represent a predetermined spanning tree $T^0$. For each bridge $i, j \in \{1, 2, \ldots, 10\}$, its traveling time $\xi_i$ is assumed to be a linear uncertain variable with distribution $\mathcal{L}(200 - x_i, 200 - x_i + d_i)$, where $x_i$ is a bandwidth parameter with an original value $c_i$, and $d_i$ is a constant. The values of $c_i$, $d_i$, $i \in E$, are listed in Table 1.

![Figure 2. Uncertain graph for computational experiments](image)

<table>
<thead>
<tr>
<th>Edge $i$</th>
<th>Original Parameter $c_i$</th>
<th>Parameter Value $d_i$</th>
<th>Uncertain Weight $\xi_i(x_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120</td>
<td>25</td>
<td>$\mathcal{L}(200 - x_1, 225 - x_1)$</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>15</td>
<td>$\mathcal{L}(200 - x_2, 215 - x_2)$</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>10</td>
<td>$\mathcal{L}(200 - x_3, 210 - x_3)$</td>
</tr>
<tr>
<td>4</td>
<td>150</td>
<td>35</td>
<td>$\mathcal{L}(200 - x_4, 235 - x_4)$</td>
</tr>
<tr>
<td>5</td>
<td>150</td>
<td>35</td>
<td>$\mathcal{L}(200 - x_5, 235 - x_5)$</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
<td>20</td>
<td>$\mathcal{L}(200 - x_6, 220 - x_6)$</td>
</tr>
<tr>
<td>7</td>
<td>60</td>
<td>20</td>
<td>$\mathcal{L}(200 - x_7, 220 - x_7)$</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>15</td>
<td>$\mathcal{L}(200 - x_8, 215 - x_8)$</td>
</tr>
<tr>
<td>9</td>
<td>170</td>
<td>40</td>
<td>$\mathcal{L}(200 - x_9, 240 - x_9)$</td>
</tr>
<tr>
<td>10</td>
<td>130</td>
<td>30</td>
<td>$\mathcal{L}(200 - x_{10}, 230 - x_{10})$</td>
</tr>
</tbody>
</table>

Table 1. Parameter values in Figure 2

According to model (11), if we want to minimize the total modification of bandwidths with a given confidence level $\alpha = 0.8$ so as to diminish the total cost of reconstruction, we have the following uncertain chance-constrained programming problem

$$
\begin{align*}
\min_x & \quad \sum_{i=1}^{10} |x_i - c_i| \\
\text{subject to:} & \quad \mathcal{M}\{\xi_i(x_i) - \xi_j(x_j) \leq 0 \} \geq 0.8,
\end{align*}
$$

where the non-tree edge set $E \setminus T^0 = \{6, 7, 8, 9, 10\}$. Using the uncertain path optimality condition II, it can be transformed into

$$
\begin{align*}
\min_x & \quad \sum_{i=1}^{10} |x_i - c_i| \\
\text{subject to:} & \quad \Phi_j^{-1}(x_i, 0.8) - \Phi_j^{-1}(x_j, 0.2) \leq 0,
\end{align*}
$$

where $j = 6, 7, \ldots, 10, i \in P_j$.

Moreover, by (4), $\Phi_j^{-1}(x_i, 0.8) = 200 + 0.8d_i - x_i$ and $\Phi_j^{-1}(x_j, 0.2) = 200 + 0.2d_j - x_j$. Hence

$$
\Phi_j^{-1}(x_i, 0.8) = 200 + 0.8d_i - x_i = 200 + 0.8d_i - x_i.
$$

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\[ \Phi^{-1}_i(x_i, 0.8) - \Phi^{-1}_j(x_j, 0.2) = -x_i + x_j + 0.8d_i - 0.2d_j. \]  \tag{21}

Consequently, it becomes

\[
\begin{aligned}
\min_{i=1}^{10} & \sum (y^+_i + y^-_i) \\
\text{subject to:} & \\
\end{aligned}
\]

\[
y^+_i - y^-_i - y^+_j + y^-_j \leq 0.2d_j - 0.8d_i + c_i - c_j, \quad j = 6, 7, \ldots, 10, \ i \in P_j \\
y^+_i, y^-_i \geq 0.
\]  \tag{22}

Using MATLAB optimization package, an optimal solution is found at

\[ x^* = (120, 68, 103, 150, 60, 60, 40, 103, 128) \]

with an objective value 170.

The predetermined confidence level \( \alpha \) is an important parameter in the formulation. The numerical experiments are further considered for different confidence levels in order to investigate the influence of this parameter. The settings and results are summarized in Table 2 including the optimal solutions and the minimum objective values.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
<th>( x_6 )</th>
<th>( x_7 )</th>
<th>( x_8 )</th>
<th>( x_9 )</th>
<th>( x_{10} )</th>
<th>OBJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>120</td>
<td>72</td>
<td>103</td>
<td>150</td>
<td>150</td>
<td>60</td>
<td>60</td>
<td>40</td>
<td>98</td>
<td>121</td>
<td>185</td>
</tr>
<tr>
<td>0.8</td>
<td>120</td>
<td>68</td>
<td>103</td>
<td>150</td>
<td>150</td>
<td>60</td>
<td>60</td>
<td>40</td>
<td>103</td>
<td>128</td>
<td>170</td>
</tr>
<tr>
<td>0.7</td>
<td>120</td>
<td>65</td>
<td>102</td>
<td>150</td>
<td>150</td>
<td>60</td>
<td>60</td>
<td>40</td>
<td>107</td>
<td>130</td>
<td>160</td>
</tr>
<tr>
<td>0.6</td>
<td>120</td>
<td>61</td>
<td>101</td>
<td>150</td>
<td>150</td>
<td>60</td>
<td>60</td>
<td>40</td>
<td>111</td>
<td>130</td>
<td>151</td>
</tr>
<tr>
<td>0.5</td>
<td>120</td>
<td>58</td>
<td>99</td>
<td>150</td>
<td>150</td>
<td>60</td>
<td>60</td>
<td>40</td>
<td>114</td>
<td>130</td>
<td>143</td>
</tr>
</tbody>
</table>

It is clear that when the confidence level \( \alpha \) decreases from 0.9 to 0.5, the corresponding optimal objective value, i.e., the minimum reconstruction cost, decreases accordingly. That is, the higher confidence level the decision-maker demands, the more cost is needed.

### 6. Conclusion

As a special type of inverse optimization problems, the inverse spanning tree problem has been extensively discussed in the literature. However, the applications of the inverse spanning tree problem encountered in practice usually involve some uncertain issues so that the edge weights cannot be explicitly determined.

An uncertain chance-constrained programming model is proposed in this paper to handle the inverse spanning tree problem where the edge weights are assumed to be uncertain variables. It is shown that the notion of the uncertain \( \alpha \)-minimum spanning tree can be characterized by a set of constraints on non-tree edges and their corresponding tree paths. By this characterization, referred to as the path optimality condition, the proposed uncertain chance-constrained programming model can be transformed into a deterministic programming model. Furthermore, when the edge weights are linear uncertain variables, it further reduces to a linear programming model and hence can be solved efficiently.

### 7. Acknowledgments

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### 8. References

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