A Modified Uncertain Entailment Model

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Abstract. Uncertain entailment model, presented by Liu in 2009, is a methodology for calculating the truth values of all uncertain formulae from an arbitrary subfamily of uncertain formulae with known truth values. In this paper, we find that the uncertain entailment model works only under some additional conditions. To sketch these conditions, a new notion of uncertain measure-evaluation (U-evaluation for short) is introduced, and a consistent theory based on it is presented. Then a modified (or more concrete) uncertain entailment model is given. Also, a practical example is provided to illustrate the effectiveness of the model.

Keywords: Nonclassical logic, uncertain logic, uncertain entailment

1. Introduction

It is well known that uncertainty usually exists for many problems in real life. First, in 1933, Kolmogoroff presented probability theory dealing with randomness. In 1965, aiming at fuzziness, Zadeh proposed fuzzy set theory via membership function. In order to deal with human uncertainty, uncertainty theory was proposed by Liu [4] in 2007 and refined by Liu [5] in 2010. Nowadays, uncertainty theory has been applied to uncertain programming (Gao[2], Zhang[14,15]), uncertain logic (Li and Liu [7], Chen[1], Liu [9]), uncertain process (Yao[3],Zhang[13]), Uncertain Differential Equations (Yao[12], Sheng[10]) etc. In uncertain logic [7], each formula is regarded as an uncertain variable, and its truth value is defined as the uncertain measure of that formula is true. In 2011, Chen and Ralescu [1] presented a Truth Value Theorem which offered a methodology to calculate the truth values of uncertain formulae when the truth values of some uncertain propositions are given. In [9], Liu presented an uncertain entailment model which gave a methodology to calculate the truth values of uncertain formulae via the maximum uncertainty principle when the truth values of a subfamily of uncertain formulae are given. Unfortunately, the authors find that the uncertain entailment model is rough. It works only under some additional conditions. Therefore, the paper further studies the model.

The remainder of this paper is organized as follows. In Section 2, some basic concepts and results about uncertainty theory and uncertain logic are recalled. In Section 3, a new notion of uncertain measure-evaluation (U-evaluation for short) is introduced, and a consistent theory based on it is presented. Then a modified (or more concrete) uncertain entailment model is given. It seems that this new model is more convenient for application. In Section 4, a practical example is provided to illustrate the effectiveness of the new model.

2. Preliminaries

In this section, we will introduce some useful definitions and results about uncertainty theory and uncertain logic.

Definition 2.1 (Liu [4]). Let $\Gamma$ be a nonempty set, and $\mathcal{L}$ a $\sigma$-algebra over $\Gamma$. Each element $\Lambda \in \mathcal{L}$ is called an event. A set function $\mathcal{M} : \mathcal{L} \rightarrow [0, 1]$ is called an uncertain measure if it satisfies the following three axioms:

Axiom 1. (Normality Axiom) $\mathcal{M}(\Gamma) = 1$;
Theorem 2.2 (Liu [5]).

**Axiom 2.** (Duality Axiom) \( \mathcal{M}(A) + \mathcal{M}(A^c) = 1 \) for any event \( A \);

**Axiom 3.** (Subadditivity Axiom) For every countable sequence of events \( \{A_i\} \), we have
\[
\mathcal{M} \left( \bigcup_{i=1}^{\infty} A_i \right) \leq \sum_{i=1}^{\infty} \mathcal{M}(A_i).
\]

The triplet \((\Gamma, \mathcal{L}, \mathcal{M})\) is called an uncertainty space. In order to obtain an uncertain measure of compound event, a product uncertain measure was defined by Liu [6], thus producing the fourth axiom of uncertainty theory:

**Axiom 4** (Product Axiom) Let \((\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)\) be an uncertainty space for \( k = 1, 2, \ldots, n \). Then the product uncertain measure on \( \Gamma \) is an uncertain measure on the product \( \sigma \)-algebra \( \mathcal{M} \left( \prod_{k=1}^{n} A_k \right) = \min_{1 \leq k \leq n} \mathcal{M}_k \{ A_k \} \).

**Definition 2.2** (Liu [5]). An uncertain variable is a measurable function \( X \) from an uncertainty space \((\Gamma, \mathcal{L}, \mathcal{M})\) to the set of real numbers, i.e., for any Borel set \( B \) of real numbers, the set \( \{ X \in B \} = \{ \gamma \in \Gamma | X(\gamma) \in B \} \) is an event.

**Definition 2.3** (Liu [5]). The uncertain variables \( X_1, X_2, \ldots, X_m \) are said to be independent if
\[
\mathcal{M} \left( \bigcap_{i=1}^{m} (X_i \in B_i) \right) = \min_{1 \leq k \leq n} \mathcal{M} \{ X_i \in B_i \},
\]
for any Borel sets \( B_1, B_2, \ldots, B_m \) of real numbers.

**Theorem 2.1** (Liu [5]). Uncertain measure \( \mathcal{M} \) is a monotone increasing set function. That is, for any events \( A_1 \subset A_2 \), we have \( \mathcal{M}(A_1) \leq \mathcal{M}(A_2) \).

**Theorem 2.2** (Liu [5]). If the uncertain variables \( X_1, X_2, \ldots, X_m \) are independent, then
\[
\mathcal{M} \left( \bigcup_{i=1}^{m} (X_i \in B_i) \right) = \max_{1 \leq i \leq m} \mathcal{M} \{ X_i \in B_i \},
\]

**Definition 2.4** (Li and Liu [7]). An uncertain proposition \( \xi \) is a statement whose truth value is quantified by an uncertain measure. An uncertain formula \( X \) is a finite sequence of uncertain propositions and connective symbols which must make sense.

In fact, an uncertain formula \( X \) is an uncertain variable taking value \( 0 \) or \( 1 \), where \( X = 1 \) means \( X \) is true and \( X = 0 \) means \( X \) is false.

**Definition 2.5** (Li and Liu [7]). Some uncertain formulae are called independent if they are independent uncertain variables.

**Definition 2.6** (Li and Liu [7]). Let \( X \) be an uncertain formula. Then the truth value of \( X \) is defined as the uncertain measure of that \( X \) is true, i.e.,
\[
T(X) = \mathcal{M}(X = 1).
\]

**Theorem 2.3** (Li and Liu [7]). Let \( X \) and \( Y \) be independent uncertain formulae. Then
\[
T(X \land Y) = T(X) \land T(Y), T(X \lor Y) = T(X) \lor T(Y).
\]

**Definition 2.7** (Wang[11]). The function \( f : \{0, 1\}^n \rightarrow \{0, 1\} \) is called a Boole function.

For example, if \( f(0, 1) = f(1, 1) = f(0, 1) = 1, f(1, 0) = 0 \), then \( f \) is a binary Boole function.

Let \( X \) be a formula containing uncertain propositions \( \xi_1, \xi_2, \ldots, \xi_n \). Since \( \xi_1, \xi_2, \ldots, \xi_n \) are all uncertain variable taking value \( 0 \) or \( 1 \), from classical logic [11], there is a Boole function \( f_X : \{0, 1\}^n \rightarrow \{0, 1\} \) such that \( X = 1 \) if \( f_X(x_1, x_2, \ldots, x_n) = 1 \). By Definition 2.5 we can know that \( f_X(x_1, x_2, \ldots, x_n) \equiv 1 \) when \( T(X) = 1 \) and \( f_X(x_1, x_2, \ldots, x_n) \equiv 0 \) when \( T(X) = 0 \).

**Theorem 2.4** (Truth Value Theorem) (Chen and Ralescu [1]). Assume that \( \xi_1, \xi_2, \ldots, \xi_n \) are independent uncertain propositions with truth values \( a_1, a_2, \ldots, a_n \), respectively. If \( X \) is an uncertain formula containing \( \xi_1, \xi_2, \ldots, \xi_n \) with Boole function \( f_X \), then
\[
T(X) = \begin{cases} 
\sup_{1 \leq i \leq n} \min v_i(x_i), & \text{if } f_X(x_1, x_2, \ldots, x_n) = 1 \land 0 \leq v_i(x_i) < 0.5; \\
1 - \sup_{1 \leq i \leq n} \min v_i(x_i), & \text{if } f_X(x_1, x_2, \ldots, x_n) = 1 \land v_i(x_i) \geq 0.5;
\end{cases}
\]
where \( x_i \) takes value either \( 0 \) or \( 1 \), and
\[
v_i(x_i) = \begin{cases} 
a_i, & \text{if } x_i = 1, \\
1 - a_i, & \text{if } x_i = 0,
\end{cases}
\]
for \( i = 1, 2, \ldots, n \), respectively.

**Theorem 2.5** (Entailment Model [9]). Let \( \xi_1, \xi_2, \ldots, \xi_n \) be independent uncertain propositions with unknown truth values \( a_1, a_2, \ldots, a_n \), respectively. Assume \( X_1, X_2, \ldots, X_m \) are uncertain formulae containing \( \xi_1, \xi_2, \ldots, \xi_n \)
with known truth values $\beta_1, \beta_2, \ldots, \beta_n$, respectively. Then the truth value $T(X)$ of an additional uncertain formula $X$ containing $\xi_1, \xi_2, \ldots, \xi_n$ solves

$$
\begin{equation}
\begin{cases}
\min |T(X) - 0.5| \\
T(X_i) = \beta_i, i = 1, 2, \ldots, m \\
0 \leq \alpha_j \leq 1, j = 1, 2, \ldots, n
\end{cases}
\end{equation}
$$

where $T(X_i), i = 1, 2, \ldots, m, T(X)$ are functions of $\alpha_1, \alpha_2, \ldots, \alpha_n$ via Truth Value Theorem. From the definition of truth value of uncertain formula, it should be pointed out that $\beta_i, i = 1, 2, \ldots, m$ in the model (1) are not arbitrary real numbers in $[0, 1]$. The uncertain entailment model may become void if we do not give some restriction conditions on the truth values of $X_i (i = 1, 2, \ldots, m)$ as the following example shows.

**Example 2.1.** Let $\xi_1, \xi_2, \xi_3$ be three independent uncertain propositions and $X_1 = \xi_1 \land \xi_2, X_2 = \xi_1 \rightarrow (\xi_1 \rightarrow \xi_2)$ and $X_3 = (\xi_1 \rightarrow \xi_2) \lor \xi_3$. Also, let $\beta_1 = 0.4, \beta_2 = 0.7$ and $\beta_3 = 0.5$. By the Truth Value Theorem, the conditions $T(X_i) = \beta_i (i = 1, 2, 3)$ in uncertain entailment model are written as follows:

1. $(\alpha_1 \land \alpha_2 \land \alpha_3) \lor (\alpha_1 \land \alpha_2 \land (1 - \alpha_3)) = 0.4$, $\alpha_1 \land \alpha_2 \land (1 - \alpha_3) = 0.3$, $\alpha_1 \land \alpha_2 \land (1 - \alpha_3) \lor (\alpha_1 \land (1 - \alpha_2) \land (1 - \alpha_3)) = 0.5$, where $T(\xi_i) = \alpha_i, i = 1, 2, 3$. Obviously, the second and the third equation can not hold at the same time. That means the uncertain entailment model dose not work in this case. Therefore, we need to give some restriction conditions of the model (1). In order to get this purpose, we give some interesting laws of truth values of uncertain formulae at last in this section.

**Theorem 2.6.** For any uncertain formulae $X$ and $Y$, we have

1. $T(X) = 1$ if $f_X = \equiv 1$; $T(X) + T(\neg X) = 1$; $T(Y) = f_X \land f_Y$; $T(X) \leq T(Y)$ if $f_X \rightarrow f_Y \equiv 1$; $T(X) + T(Y) - 1 \leq T(X \land Y) = T(Y \land X) \leq T(Y) \lor T(X)$;
2. $T(X \lor T(Y) \leq T(X \lor Y) \leq T(X) + T(Y)$;
3. $(1 - T(X)) \lor T(Y) \leq T(X \rightarrow Y) \leq 1 - T(X) + T(Y)$.

**Proof.** In [7], (ii), (v) and (vi) have been proved.

(i) It follows from Theorem 2.4 that $T(X) = 1$.

(ii) If $f_X \equiv f_Y$, then $\{X = 1\} = \{Y = 1\}$ and so $T(X) = M\{X = 1\} = M\{Y = 1\} = T(Y)$.

(iv) Let $f_{X \rightarrow Y} \equiv 1$. Then $X = 1$ implies $Y = 1$ from classical logic [11]. Which means $\{X = 1\} \subseteq \{Y = 1\}$. By the monotonicity of uncertain measure, we have $T(X) = M\{X = 1\} \leq M\{Y = 1\} = T(Y)$.

(vi) From Theorem 2.6 (ii) and (vi) we obtain $T(X \rightarrow Y) = T(X) + T(Y)$

and $T(X \rightarrow Y) = T(X \lor Y) \geq T(X) \lor T(Y)$

Then (vii) has been proved.

3. A Modified Uncertain Entailment Model

In this section, we shall introduce some new concepts of uncertain measure-evaluation and consistent theory. Then we present a modified uncertain entailment model.

**Definition 3.1.** Let $S = \{\xi_1, \xi_2, \ldots, \xi_n\}$ be a set of independent uncertain propositions. A $(\rightarrow, \lor, \land)$-type free algebra generated by $S$ is called an uncertain logic system and is denoted by $F(S)$. A subset $\Gamma \subseteq F(S)$ is called a theory of $F(S)$.

It is evident that if the truth values of independent uncertain propositions $\xi_1, \xi_2, \ldots, \xi_n$ are given, then the truth values of all formulae in $F(S)$ are uniquely determined via Truth Value Theorem. This leads the following notion.

**Definition 3.2.** Let $S = \{\xi_1, \xi_2, \ldots, \xi_n\}$ be a set of independent uncertain propositions. A mapping $T : F(S) \rightarrow [0, 1]$ is called an uncertain measure-evaluation ($U$-evaluation for short) on the system $F(S)$ if it satisfies the Truth Value Theorem, i.e., for each uncertain formula $X \in F(S), T(X)$ is determined by $T(\xi_1), T(\xi_2), ..., T(\xi_n)$ via Truth Valued Theorem.

**Definition 3.3.** Let $S = \{\xi_1, \xi_2, \ldots, \xi_n\}$ be a set of independent uncertain propositions, $\Gamma = \{X_1, \ldots, X_m\} \subseteq F(S)$, and $T^* : \Gamma \rightarrow [0, 1]$ be a mapping. $\Gamma$ is said to be consistent theory with $T^*$ if there exits an $U$-evaluation $T$ on $F(S)$ such that $T = T^*$ on $\Gamma$ (in this case, $T$ is also called an expansion of $T^*$), otherwise, $\Gamma$ is said to be non-consistent theory with $T^*$.

From Definition 3.3 and Theorem 2.6 we have the following theorem.
Theorem 3.1. Let $\Gamma = \{X_1, ..., X_m\} \subseteq F(S)$ be a consistent theory with mapping $T^*$. Then the following laws of truth values hold:

(i) $T^*(X_i) = 1$ if $X_i \in \Gamma$ and $f_{X_i} \equiv 1, T^*(X_i) = 0$ if $X_i \notin \Gamma$ and $f_{X_i} \equiv 0, i \in \{1, 2, ..., m\}$,

(ii) $T^*(X_i) = T^*(X_j)$ if $f_{X_i} = 1 \iff f_{X_j} = 1$, for any $X_i, X_j \in \Gamma, \ i \neq j, i, j \in \{1, 2, ..., m\}$,

(iii) $T^*(X_i) + T^*(X_j) = 1$ if $f_{X_i} = 1 \iff f_{X_j} = 0$, for any $X_i, X_j \in \Gamma, \ i \neq j, i, j \in \{1, 2, ..., m\}$,

(iv) $T^*(X_i) \leq T^*(X_j)$ if $f_{X_i} \rightarrow X_j \equiv 1$, for any $X_i, X_j \in \Gamma, \ i \neq j, i, j \in \{1, 2, ..., m\}$,

(v) $T^*(X_i) + T^*(X_j) - 1 \leq T^*(X_k)$ if $X_i = X_j \land X_k$, for any $X_i, X_j, X_k \in \Gamma, \ i, j, t \in \{1, 2, ..., m\}$,

(vi) $T^*(X_i) \leq T^*(X_j) + T^*(X_k)$ if $X_i = X_j \lor X_k$, for any $X_i, X_j, X_k \in \Gamma, \ i, j, t \in \{1, 2, ..., m\}$.

\[ \left\{ \begin{array}{l} \min \{T(X) - 0.5\} \\ \text{Subject to Eq(3.1)} \end{array} \right. \]

\[ \begin{align*} \bigwedge_{1 \leq i \leq n} (\bigwedge_{1 \leq i \leq n} \alpha_{x_i}) = \beta_1; \\
& \vdots \\
& \bigwedge_{1 \leq i \leq n} (\bigwedge_{1 \leq i \leq n} \alpha_{x_i}) = \beta_{i-1}; \\
& \bigwedge_{1 \leq i \leq n} (\bigwedge_{1 \leq i \leq n} \alpha_{x_i}) = 0.5; \\
& \vdots \\
& \bigwedge_{1 \leq i \leq n} (\bigwedge_{1 \leq i \leq n} \alpha_{x_i}) = 1 - \beta_{t+1}; \\
& \vdots \\
& \bigwedge_{1 \leq i \leq n} (\bigwedge_{1 \leq i \leq n} \alpha_{x_i}) = 1 - \beta_m; \end{align*} \]

where $T$ is an extension of $T^*$ and $\alpha_{x_i} = T(\xi_i)$ if $x_i = 1$; $\alpha_{x_i} = 1 - T(\xi_i)$ if $x_i = 0$. Note that the existence of $T$ is equivalent to the consistence of $\Gamma$ with $T^*$. Then we obtain a modified uncertain entailment model.

Example 3.1. Let $\xi_1, \xi_2$ be independent uncertain propositions and $\Gamma = \{\xi_1, \xi_2 \lor \xi_2\}$. It is easily seen that the following fact:

(i) The mapping $T^* : \Gamma \rightarrow [0, 1]$, defined by $T^*(\xi_1) = 0.4$ and $T^*(\xi_1 \land \xi_2) = 0.3$, is consistent with $\Gamma$.

(ii) The mapping $T^* : \Gamma \rightarrow [0, 1]$, defined by $T^*(\xi_1) = 0.4, T^*(\xi_1 \land \xi_2) = 0.5$, is not consistent with $\Gamma$.

(iii) The mapping $T^* : \Gamma \rightarrow [0, 1]$, defined by $T^*(\xi_1 \land \xi_2) = T^*(\xi_1) = 0.3$, is consistent with $\Gamma$.

And the family of mappings $T : F(S) \rightarrow [0, 1]$, determined by $T(\xi_1) = 0.3, T(\xi_2) = \alpha \in [0.3, 1]$ via Truth Value Theorem, are all extensions of $T^* \lor T$.

The above example shows that for a theory $\Gamma \subseteq F(S)$ and a mapping $T^* : \Gamma \rightarrow [0, 1]$, we have none, one or many $U$-evaluation $T$ which extending $T^*$ w.r.t $\Gamma$.

Let $S = \{\xi_1, \xi_2, ..., \xi_n\}$ be a set of independent uncertain propositions containing uncertain propositions $\xi_1, \xi_2, ..., \xi_n$, $\Gamma = \{X_1, X_2, ..., X_m\} \subseteq F(S)$ be consistent theory with $T^*: \Gamma \rightarrow [0, 1]$. Suppose that $T^*(X_i) = \beta_i, i = 1, 2, ..., m$. Then $\beta_i \in [0, 0.5]$ or $\beta_i = 0.5$ or $\beta_i \in (0.5, 1]$. Without loss of generality, we assume that $\beta_1, \beta_2, ..., \beta_t \in [0, 0.5], \beta_1 = 0.5, \beta_{t+1}, ..., \beta_m \in (0.5, 1]$. Then the equation $T^*(X_i) = \beta_i (i = 1, 2, ..., m)$ in Liu’s uncertain entailment model can be rewritten as the following Eq(3.1):

\[ \bigwedge_{1 \leq i \leq n} (\bigwedge_{1 \leq i \leq n} \alpha_{x_i}) = \beta_1; \]

\[ \vdots \]

\[ \bigwedge_{1 \leq i \leq n} (\bigwedge_{1 \leq i \leq n} \alpha_{x_i}) = \beta_{i-1}; \]

\[ \bigwedge_{1 \leq i \leq n} (\bigwedge_{1 \leq i \leq n} \alpha_{x_i}) = 0.5; \]

\[ \vdots \]

\[ \bigwedge_{1 \leq i \leq n} (\bigwedge_{1 \leq i \leq n} \alpha_{x_i}) = 1 - \beta_{t+1}; \]

\[ \vdots \]

\[ \bigwedge_{1 \leq i \leq n} (\bigwedge_{1 \leq i \leq n} \alpha_{x_i}) = 1 - \beta_m; \]

\[ \left\{ \begin{array}{l} \min \{T(X) - 0.5\} \\ \text{Subject to Eq(3.1)} \end{array} \right. \]

4. Example of Uncertain Entailment

In this section, we shall show the effectiveness of the modified uncertain entailment model by an example on
5. Conclusions

we conclude that
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