An Uncertain Random Programming Model for Project Scheduling Problem

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Abstract: Project scheduling problem is to determine the resource allocation schedule for the trade-off between the project cost and the completion time. In this paper, project scheduling problem in the environment with uncertainty and randomness simultaneously is considered. In detail, the concepts of uncertain variable and uncertain random variable are introduced. Based on some concepts and theorems of chance theory, an uncertain random project scheduling model is built. For some special case, the proposed uncertain random programming model is transformed to a crisp mathematical programming model. Besides, uncertain random simulation techniques and genetic algorithm are integrated into a hybrid intelligent algorithm for searching the quasi-optimal schedule.

Keywords: project scheduling, uncertain random programming, uncertain programming, uncertainty theory, chance theory

1 Introduction

Project scheduling problem (PSP) is to determine the resource allocation schedule for the trade-off between the project cost and the completion time. In 1960s, Kelley [17, 18] originally proposed a deterministic project scheduling problem, which was continuously discussed by many other researchers [4, 35].

Since in the real world the project environments are always indeterminate, probability theory was introduced into PSP, originally by Freeman [5]. After that, Charnes and Cooper [1] employed chance constrained programming philosophy into stochastic project scheduling problem (SPSP). Golenko-Ginzburg and Gonik [7] built a stochastic project scheduling model with the objective of minimizing expected completion time. Ke and Liu [9] established three stochastic models to solve PSP with stochastic activity duration times. Ke et al. [11] built three types of stochastic models for describing project time-cost trade-offs.

Though probability theory has been well applied in the research of PSP, the indeterminacy in real-life projects can not be explained by randomness in all cases. For the lack of statistical data, probability theory is not suitable for some projects. In 1979, Prade [32] applied fuzzy set theory into PSP. Hapke and Slowinski [8] applied simulated annealing into the fuzzy resource-constrained PSP for solving some multi-objective cases. Long and Ohsato [30] performed a fuzzy critical chain method for fuzzy resource-constrained PSP. Ke and Liu [12] designed three types of fuzzy models for PSP. Ke et al. [13] established three models for solving fuzzy time-cost trade-off problem.

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In decision-making systems with the shortage of historical data, for describing system indeterminacy, some domain experts may give belief degrees for some quantities. Traditionally, belief degree was considered as subjective probability or fuzzy concept. However, since human beings usually overweight unlikely events, probability theory is no longer valid for belief degree. If possibility measure is applied for describing belief degree, it may lead to some counterintuitive results in some cases. Consider some knowledge via human language like “old age”, “low speed”, “approximately 60kg”, and “roughly 100km”. For instance, to measure “roughly 100km” by possibility measure, we may see that the distance is “exactly 100km” with belief degree 1, and is “not 100km” with belief degree 1 as well, which means “exactly 100km” and “not 100km” are equally likely. It is unbelievable for anyone with common sense. Hence, these types of imprecise quantities can not be described as fuzziness. When indeterminate phenomena behave neither randomness nor fuzziness, we need a new tool to illustrate it. To model such imprecise quantities, Liu founded an uncertainty theory in 2007 [22] and refined it in 2010 [24]. Uncertainty theory is a branch of axiomatic mathematics for modeling human uncertainty, which has been deeply developed in many fields such as uncertain calculus [3, 23, 27, 38], uncertain risk analysis [25], and uncertain logic [26]. Since the inception in 2007, uncertainty theory has been successfully applied in many uncertain decision-making situations, e.g., option pricing [2, 31], facility location [6], transportation problem [34], and portfolio selection [41]. Especially, Zhang and Chen [39] built a new uncertain programming model for PSP. Ke et al. [16] built two models for optimizing project time-cost trade-off based on uncertain measure.

Sometimes subjective and objective indeterminacies may exist in the same project. Some activity duration times may have statistical data enough to determine probability distributions, while some other activities may be short of historical data. Ke and Liu [10] introduced random fuzzy theory, founded by Liu [21], to model PSP in the complicated environment with the coexisted randomness and fuzziness. Ke et al. [14] described the complicated indeterminacy of PSP via fuzzy random variable, initiated by Kwakernaak [19, 20]. Since belief degree can not be described as subjective probability or fuzziness, when subjective and objective indeterminacies coexist in some decision systems, it is better to introduce uncertain random variable and chance theory proposed by Liu [28] to model PSP. The chance theory has been promoted in some aspects, e.g., in law of large numbers [36], and uncertain random process [37]. The uncertain random programming was presented by Liu [29]. Some work has been done on the extension of uncertain random programming, such as in multilevel programming [15], goal programming [33] and multiobjective programming [40]. In this paper, we will introduce the uncertain random programming philosophy into PSP and build an uncertain random PSP model.

The remainder of the paper is organized as follows: Section 2 introduces some concepts and theorems of uncertainty theory and uncertain random programming. Section 3 describes the uncertain random project scheduling problem in detail and builds an uncertain random expected cost minimization model. In Section 4, two uncertain random simulation techniques are proposed in detail, and a hybrid intelligent algorithm integrating the proposed uncertain random simulations and genetic algorithm (GA) is designed for solving the model. Section 5 gives a numerical experiment. Finally, some conclusions are drawn in Section 6.

2 Preliminary

In this section, we will introduce some concepts and theorems on uncertain variable and uncertain random variable.
2.1 Uncertainty Theory

Uncertainty theory, founded by Liu [22] in 2007 and refined by Liu [24] in 2010, is a branch of axiomatic mathematics for modeling human uncertainty. Let \( \Gamma \) be a nonempty set, \( \mathcal{L} \) a \( \sigma \)-algebra over \( \Gamma \), and each element \( \Lambda \) in \( \mathcal{L} \) is called an event. Uncertain measure is defined as a function from \( \mathcal{L} \) to \([0, 1]\). In detail, Liu [22] gave the concept of uncertain measure as follows:

**Definition 1** (Liu [22]) The set function \( \mathcal{M} \) is called an uncertain measure if it satisfies:

- **Axiom 1.** \( \mathcal{M}(\Gamma) = 1 \) for the universal set \( \Gamma \).
- **Axiom 2.** \( \mathcal{M}(\Lambda) + \mathcal{M}(\Lambda^c) = 1 \) for any event \( \Lambda \).
- **Axiom 3.** For every countable sequence of events \( \Lambda_1, \Lambda_2, \cdots \), we have
  \[
  \mathcal{M}\left(\bigcup_{i=1}^{\infty} \Lambda_i\right) \leq \sum_{i=1}^{\infty} \mathcal{M}(\Lambda_i).
  \]

Besides, the product uncertain measure on the product \( \sigma \)-algebra \( \mathcal{L} \) was defined by Liu [23] as follows:

**Axiom 4.** Let \((\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)\) be uncertainty spaces for \( k = 1, 2, \cdots \). The product uncertain measure \( \mathcal{M} \) is an uncertain measure satisfying

\[
\mathcal{M}\left(\prod_{k=1}^{\infty} \Lambda_k\right) = \bigwedge_{k=1}^{\infty} \mathcal{M}_k(\Lambda_k)
\]

where \( \Lambda_k \) are arbitrarily chosen events from \( \mathcal{L}_k \) for \( k = 1, 2, \cdots \), respectively.

Based on the concept of uncertain measure, we can define an uncertain variable.

**Definition 2** (Liu [22]) An uncertain variable is a measurable function \( \xi \) from an uncertainty space \((\Gamma, \mathcal{L}, \mathcal{M})\) to the set of real numbers, i.e., for any Borel set \( B \) of real numbers, the set

\[
\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\}
\]

is an event.

Sometimes, to model real-life uncertain optimization problems, it is sufficient to know the uncertainty distribution rather than the uncertain variable itself.

**Definition 3** (Liu [22]) The uncertainty distribution \( \Phi \) of an uncertain variable \( \xi \) is defined by

\[
\Phi(x) = \mathcal{M}\{\xi \leq x\}
\]

for any real number \( x \).

Liu [22] also defined the expected value of an uncertain variable as follows:

**Definition 4** (Liu [22]) Let \( \xi \) be an uncertain variable. The expected value of \( \xi \) is defined by

\[
E[\xi] = \int_{0}^{+\infty} \mathcal{M}\{\xi \geq r\}dr - \int_{-\infty}^{0} \mathcal{M}\{\xi \leq r\}dr
\]

provided that at least one of the above two integrals is finite.
Theorem 1 (Liu [24]) Let $\xi$ be an uncertain variable with regular uncertainty distribution $\Phi$. If the expected value exists, then

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha)d\alpha.$$ 

For instance, let $\xi \sim L(a, b)$ be a linear uncertain variable. Then its inverse uncertainty distribution is $\Phi^{-1}(\alpha) = (1 - \alpha)a + ab$, and its expected value is

$$E[\xi] = \int_0^1 ((1 - \alpha)a + ab)d\alpha = \frac{a + b}{2}.$$

2.2 Uncertain Random Variable

Based on the definitions of uncertain variable and random variable, the concept of an uncertain random variable can be given as follows:

Definition 5 (Liu [28]) An uncertain random variable is a function $\xi$ from a probability space $(\Omega, \mathcal{A}, Pr)$ to a collection of uncertain variables such that

$$M\{\xi(\omega) \in B\}$$

is a measurable function of $\omega$ for any Borel set $B$ of real numbers.

Example 1 An uncertain variable is a special uncertain random variable.

Example 2 A random variable is a special uncertain random variable since any real value is a special uncertain variable.

Example 3 Let $\xi$ be a random variable, and $\eta$ an uncertain variable. Then the sum $\zeta = \xi + \eta$ is an uncertain random variable.

For measuring uncertain random event, we introduce the following definitions:

Definition 6 (Liu [28]) Let $\xi$ be an uncertain random variable, and $B$ a Borel set of real numbers. Then the chance measure of uncertain random event $\xi \in B$ is defined by

$$Ch\{\xi \in B\} = \int_0^1 Pr\{\omega \in \Omega| M\{\xi(\omega) \in B\} \geq r\}dr.$$ 

Liu [28] also stated that the chance measure $Ch\{\xi \in B\}$ is in fact the expected value of $M\{\xi(\cdot) \in B\}$, i.e.,

$$Ch\{\xi \in B\} = E[M\{\xi(\cdot) \in B\}]$$

since $M\{\xi(\omega) \in B\}$ is a measurable function of $\omega$ and then $M\{\xi(\cdot) \in B\}$ is a random variable.

Definition 7 (Liu [28]) Let $\xi$ be an uncertain random variable. Then the chance distribution of $\xi$ is defined by

$$\Phi(x) = Ch\{\xi \leq x\}$$

for any $x \in R$.

Liu [28] also proved that a function $\Phi : R \rightarrow [0, 1]$ is a chance distribution if and only if it is a monotone increasing function except $\Phi(x) \equiv 0$ and $\Phi(x) \equiv 1$.

Definition 8 (Liu [28]) Let $\xi$ be an uncertain random variable. Then its expected value is defined by

$$E[\xi] = \int_0^{+\infty} Ch\{\xi \geq r\}dr - \int_{-\infty}^0 Ch\{\xi \leq r\}dr$$

provided that at least one of the two integrals is finite.
Assume the uncertain random variable $\xi$ has a chance distribution $\Phi$. When the expected value of $\xi$ exists, Liu [28] showed that

$$E[\xi] = \int_0^{+\infty} (1 - \Phi(x))dx - \int_{-\infty}^0 \Phi(x)dx.$$  

For some special case of uncertain random variable, Liu [29] presented an expected value formula of function of uncertain random variables.

**Theorem 2** (Liu [29]) Let $\eta_1, \eta_2, \ldots, \eta_m$ be independent random variables with probability distributions $\Psi_1, \Psi_2, \ldots, \Psi_m$, and $\tau_1, \tau_2, \ldots, \tau_n$ be independent uncertain variables with uncertainty distributions $\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n$, respectively. Then the uncertain random variable $\xi = f(\eta_1, \eta_2, \ldots, \eta_m, \tau_1, \tau_2, \ldots, \tau_n)$ has an expected value

$$E[\xi] = \int_{R^m} \int_0^1 f(y_1, \ldots, y_m, \Upsilon_1^{-1}(\alpha), \ldots, \Upsilon_n^{-1}(\alpha))d\alpha d\Psi_1(y_1) \ldots d\Psi_m(y_m)$$

provided that $f(\eta_1, \eta_2, \ldots, \eta_m, \tau_1, \tau_2, \ldots, \tau_n)$ is a strictly increasing function with respect to $\tau_1, \tau_2, \ldots, \tau_n$.

**Theorem 3** (Liu [29]) Let $\eta$ be a random variable and $\tau$ be an uncertain variable. Then

$$E[\eta + \tau] = E[\eta] + E[\tau], \quad E[\eta \tau] = E[\eta]E[\tau].$$

**Theorem 4** (Liu [29]) Assume that $\eta_1$ and $\eta_2$ are (not necessarily independent) random variables, $\tau_1$ and $\tau_2$ are independent uncertain variables, and $f_1$ and $f_2$ are measurable functions. Then

$$E[f_1(\eta_1, \tau_1) + f_2(\eta_2, \tau_2)] = E[f_1(\eta_1, \tau_1)] + E[f_2(\eta_2, \tau_2)].$$

### 2.3 Uncertain Random Programming

Assume that $x$ is a decision vector, and $\xi$ is an uncertain random vector. Let the constraints be defined by $g_i(x, \xi) \leq 0$, where $g_i(x, \xi)$ are vector-valued functions, $i = 1, 2, \ldots, l$. Since $\xi$ is an uncertain random vector, the uncertain random constraints cannot always hold completely. It is naturally desired for the decision-maker to satisfy the constraints with some confidence levels $\alpha_1, \alpha_2, \ldots, \alpha_l$ as follows,

$$\text{Ch}\{g_i(x, \xi) \leq 0\} \geq \alpha_i, \; i = 1, 2, \ldots, l.$$  

Let the objective function be defined by $f(x, \xi)$. The decision-maker may want to minimize its expected value $\min_x E[f(x, \xi)]$. To obtain a decision with minimum expected objective subject to some chance constraints, Liu [29] proposed the following uncertain random programming model,

$$\left\{ \begin{array}{l} \min_x E[f(x, \xi)] \\ \text{subject to:} \\ \text{Ch}\{g_i(x, \xi) \leq 0\} \geq \alpha_i, \; i = 1, 2, \ldots, l. \end{array} \right.$$  

Liu [29] also discussed the equivalence transformation to crisp mathematical programming for some special type of uncertain random programming model.

**Theorem 5** (Liu [29]) Let $\eta_1, \eta_2, \ldots, \eta_m$ be independent random variables with probability distributions $\Psi_1, \Psi_2, \ldots, \Psi_m$, and $\tau_1, \tau_2, \ldots, \tau_n$ be independent uncertain variables with uncertainty distributions $\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n$, respectively. If $f(x, \eta_1, \eta_2, \ldots, \eta_m, \tau_1, \tau_2, \ldots, \tau_n)$ and $g_i(x, \eta_1, \eta_2, \ldots,
The indeterminacy of activity duration times derives from the variational external environment, such as the change of weather and the increase of productivity rate. For convenience, all the activity duration times are written as a vector $\xi = (\xi_{ij} : (i, j) \in A)$. It is assumed that all the costs needed for the activities are constants and described by $c_{ij}$ for all $(i, j) \in A$. Since many real-life projects, especially large-scale construction projects, always need a heavy hit of fees, most project managers turn to banks for loans. Hence, the total project cost needed here is assumed to be obtained via loans with some given interest rate $r$. We use $x = (x_1, x_2, \cdots, x_n)$ as the decision vector, where $x_i$ are assumed to be nonnegative integers and represent the allocating
times of all the loans needed for activities \((i, j), i = 1, 2, \cdots, n\), respectively. Then the problem is to find the optimal schedule \(x\) so as to minimize the project cost (actually the interest) with some completion time constraints.

The starting times of activities \((i, j)\) are denoted as \(T_i(x, \xi), i = 1, 2, \cdots, n\), respectively. For simplicity, we assume that each activity can be processed only if all the foregoing activities are finished and should be processed without interruption, and the starting time of the project is assumed as \(T_1(x, \xi) \equiv x_1 = 0\). From the assumptions, we have \(T_i(x, \xi) \geq x_1\) and \(T_i(x, \xi) \geq \max_{\{k, i\} \in A} \{T_k(x, \xi) + \xi_{ki}\}\). Hence, the starting times of activities \((i, j), i = 2, \cdots, n\), can be decided by \(T_i(x, \xi) = x_i \vee \max_{\{k, i\} \in A} \{T_k(x, \xi) + \xi_{ki}\}\), and the project completion time can be calculated by

\[
T(x, \xi) = \max_{\{k, n+1\} \in A} \{T_k(x, \xi) + \xi_{kn+1}\}. \tag{4}
\]

Applying the compound interest formula to calculate the future value of all the loans, the project cost can be written as

\[
C(x, \xi) = \sum_{(i,j) \in A} c_{ij} (1 + r)^{[(T(x, \xi) - x_i)]} \tag{5}
\]

where \([a]\) means the minimal integer larger than or equal to \(a\).

### 3.2 Uncertain Random Expected Cost Minimization Model

In real projects, tradeoffs among project goals always exist. Especially, managers tend to make necessary decisions on trade-off between project completion time and project cost. For the complex external environment, managers may want to find some optimal decision with minimum project cost subject to some project completion time constraint. Since \(\xi\) is an uncertain random vector, the time constraint \(T(x, \xi) \leq T^0\) cannot always hold completely, where \(T^0\) is the due date of the project. It is a natural choice for managers to satisfy the time constraint with at least some given confidence level \(\alpha\). That is, \(\text{Ch}[T(x, \xi) \leq T^0] \geq \alpha\). Now to minimize the expected cost of the project, we can build the uncertain random expected cost minimization model:

\[
\begin{align*}
\min E[C(x, \xi)] \\
\text{subject to:} \\
\text{Ch}[T(x, \xi) \leq T^0] \geq \alpha \\
x \geq 0, \text{ integer vector}
\end{align*}
\tag{6}
\]

where \(T^0\) is the due date of the project, \(\alpha\) is the given confidence level, and \(T(x, \xi)\) and \(C(x, \xi)\) are defined by (4) and (5), respectively.

**Remark 1:** If the uncertain random vector \(\xi\) degenerates to a random vector, then the objective function and the constraint function in the above model become

\[
E[C(x, \xi)] = \int_0^{+\infty} \Pr[C(x, \xi) \geq r] dr - \int_{-\infty}^0 \Pr[C(x, \xi) \leq r] dr,
\]

and \(\Pr[T(x, \xi) \leq T^0] \geq \alpha\), respectively. Therefore, the uncertain random expected cost minimization model becomes a stochastic expected cost minimization model.

**Remark 2:** If the uncertain random vector \(\xi\) degenerates to an uncertain vector, then the objective function and the constraint function become

\[
E[C(x, \xi)] = \int_0^{+\infty} \mathbb{M}[C(x, \xi) \geq r] dr - \int_{-\infty}^0 \mathbb{M}[C(x, \xi) \leq r] dr,
\]
and \( \mathcal{M}\{T(x, \xi) \leq T^0\} \geq \alpha \), respectively. Then the uncertain random expected cost minimization model becomes an uncertain expected cost minimization model.

We represent \( \xi \) in another way as \( \xi = (\xi_1, \xi_2, \cdots, \xi_m) \), where \( m \) is the number of the activities. Sometimes, the uncertain random vector \( \xi \) can be represented by \( \xi = (\xi_1, \xi_2, \cdots, \xi_k, \xi_{k+1}, \cdots, \xi_m) \), where \( \xi_1, \xi_2, \cdots, \xi_k \) are independent random variables with probability distributions \( \Psi_1, \Psi_2, \cdots, \Psi_k \), and \( \xi_{k+1}, \xi_{k+2}, \cdots, \xi_m \) are independent uncertain variables with uncertainty distributions \( \Upsilon_1, \Upsilon_2, \cdots, \Upsilon_{m-k} \), respectively. Since \( C(x, \xi) \) and \( T(x, \xi) \) are strictly increasing with respect to all the uncertain random activity duration times, the above model (6) can be transformed to the following crisp mathematical programming via applying Theorem 5:

\[
\begin{align*}
\min_x \int_{\mathbb{R}^k} \int_0^1 C(x, y_1, y_2, \cdots, y_k, \Psi_1^{-1}(\beta), \Psi_2^{-1}(\beta), \cdots, \Psi_{m-k}^{-1}(\beta)) d\beta d\Psi_1(y_1) \Psi_2(y_2) \cdots d\Psi_k(y_k) \\
\text{subject to:} \\
\int_{\mathbb{R}^k} H(x, y_1, y_2, \cdots, y_k) d\Psi_1(y_1) \Psi_2(y_2) \cdots d\Psi_k(y_k) \geq \alpha
\end{align*}
\]

where \( H(x, y_1, y_2, \cdots, y_k) \) is the root of

\[
T(x, y_1, y_2, \cdots, y_k, \Psi_1^{-1}(\gamma), \Psi_2^{-1}(\gamma), \cdots, \Psi_{m-k}^{-1}(\gamma)) = 0.
\]

## 4 Hybrid Intelligent Algorithm

### 4.1 Uncertain Random Simulations

In model (6), there exist two types of uncertain random functions: \( \text{Ch}\{T(x, \xi) \leq T^0\} \), and \( E[C(x, \xi)] \). To estimate these functions, we introduce uncertain random simulation techniques.

The chance \( \text{Ch}\{T(x, \xi) \leq T^0\} \) is the expected value of the random variable \( M\{T(x, \xi(\omega)) \leq T^0\} \). Then with the philosophy of stochastic simulation (Monte Carlo simulation), we can obtain the following algorithm:

**Algorithm 1:** (Uncertain Random Simulation for Chance)

**Step 1.** Set \( e = 0 \).

**Step 2.** Randomly generate \( \omega \) from the probability space according to the probability distribution.

**Step 3.** \( e \leftarrow e + M\{T(x, \xi(\omega)) \leq T^0\} \).

**Step 4.** Repeat the second and third steps \( N \) times, where \( N \) is a sufficiently large number.

**Step 5.** Return \( e/N \).

According to the concept of expected value of uncertain random variable, we can obtain the following simulation procedure for \( E[C(x, \xi)] \) via Monte Carlo simulation:

**Algorithm 2:** (Uncertain Random Simulation for Expected Value)

**Step 1.** Set \( e = 0 \).

**Step 2.** Randomly generate \( \omega \) from the probability space according to the probability distribution.
Step 3. \( e \leftarrow e + E[C(x, \xi(\omega))] \).

Step 4. Repeat the second and third steps \( N \) times, where \( N \) is a sufficiently large number.

Step 5. Return \( e/N \).

Note that in the above two algorithms, there are two uncertain functions: \( M\{T(x, \xi(\omega)) \leq T_0\} \) and \( E[C(x, \xi(\omega))] \). The interested readers can refer to Liu [24] to see the simulation processes for these two uncertain functions.

4.2 Hybrid Intelligent Algorithm

To find an optimal schedule for a project, we need to design some heuristic algorithm. Since GA is an effective method for practical optimization problems, we embed the above uncertain random simulations into GA to design a hybrid intelligent algorithm. The procedure can be summarized briefly as follows.

Algorithm 3: (Hybrid Intelligent Algorithm)

Step 1. Initialize \( \text{pop}_\text{size} \) chromosomes, where the uncertain random functions can be calculated and the feasibility can be checked by the proposed uncertain random simulation methods.

Step 2. Update the chromosomes by crossover and mutation operations, in which the feasibility of offsprings may also be checked by the proposed uncertain random simulations.

Step 3. Compute the objective values for all chromosomes and accordingly calculate the fitness of each chromosome.

Step 4. Select the chromosomes by spinning the roulette wheel.

Step 5. Run the second to fourth steps for a given number of cycles and report the best chromosome as the quasi-optimal solution.

The above algorithm procedure can be illustrated in Fig. 2.

![Algorithm Procedure](image)
Table 1: Uncertain Random Duration Times and Costs of Activities

<table>
<thead>
<tr>
<th>Activity</th>
<th>Duration Time</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,2)</td>
<td>U(9,12)</td>
<td>1500</td>
</tr>
<tr>
<td>(1,3)</td>
<td>U(5,8)</td>
<td>1800</td>
</tr>
<tr>
<td>(1,4)</td>
<td>L(8,12)</td>
<td>430</td>
</tr>
<tr>
<td>(2,5)</td>
<td>L(5,8)</td>
<td>1600</td>
</tr>
<tr>
<td>(3,6)</td>
<td>U(7,10)</td>
<td>100</td>
</tr>
<tr>
<td>(3,7)</td>
<td>L(7,11)</td>
<td>340</td>
</tr>
<tr>
<td>(3,10)</td>
<td>L(11,14)</td>
<td>1200</td>
</tr>
<tr>
<td>(4,8)</td>
<td>U(5,8)</td>
<td>6200</td>
</tr>
<tr>
<td>(5,9)</td>
<td>L(7,10)</td>
<td>450</td>
</tr>
<tr>
<td>(6,9)</td>
<td>L(5,10)</td>
<td>2100</td>
</tr>
<tr>
<td>(6,13)</td>
<td>L(7,12)</td>
<td>2800</td>
</tr>
<tr>
<td>(7,11)</td>
<td>L(8,13)</td>
<td>60</td>
</tr>
<tr>
<td>(7,14)</td>
<td>U(11,14)</td>
<td>5200</td>
</tr>
<tr>
<td>(8,11)</td>
<td>U(5,8)</td>
<td>450</td>
</tr>
<tr>
<td>(9,12)</td>
<td>L(5,9)</td>
<td>2000</td>
</tr>
<tr>
<td>(10,13)</td>
<td>U(8,11)</td>
<td>1700</td>
</tr>
<tr>
<td>(10,14)</td>
<td>U(9,14)</td>
<td>300</td>
</tr>
<tr>
<td>(10,17)</td>
<td>U(13,17)</td>
<td>150</td>
</tr>
<tr>
<td>(11,15)</td>
<td>L(4,9)</td>
<td>210</td>
</tr>
<tr>
<td>(12,16)</td>
<td>L(9,14)</td>
<td>250</td>
</tr>
<tr>
<td>(13,16)</td>
<td>L(7,9)</td>
<td>200</td>
</tr>
<tr>
<td>(14,18)</td>
<td>L(9,12)</td>
<td>300</td>
</tr>
<tr>
<td>(15,18)</td>
<td>U(8,13)</td>
<td>1100</td>
</tr>
<tr>
<td>(16,19)</td>
<td>U(8,12)</td>
<td>550</td>
</tr>
<tr>
<td>(17,19)</td>
<td>U(4,7)</td>
<td>530</td>
</tr>
<tr>
<td>(18,19)</td>
<td>L(9,14)</td>
<td>630</td>
</tr>
</tbody>
</table>

5 Numerical Experiment

Consider a project requested to be finished in 60 months, illustrated by Fig. 1. The duration times and the costs needed for the relevant activities in the project are presented in Table 1, respectively, and the monthly interest rate is set in advance as 0.6%. Note that the duration times are partly assumed to be uniformly distributed random variables denoted by U(g, h), and partly linear uncertain variables denoted by L(i, j), respectively, where g, h, i and j are given crisp numbers. Due to the environmental complexity, the project can not be ensured to be finished on time. As an alternative, the decision-maker tends to minimize the expected cost of the project and meanwhile finish the project within 60 months with the given confidence level 0.90. That is,

\[
\begin{align*}
\min & \ E[C(x, \xi)] \\
\text{subject to:} & \\
\quad & \text{Ch}\{T(x, \xi) \leq 60\} \geq 0.90 \\
\quad & x \geq 0, \text{ integer vector.}
\end{align*}
\]

The parameters of the algorithm, including the population size of one generation \( \sigma_{pop} \), the probability of mutation \( \rho_{mut} \), and the probability of crossover \( \rho_{crs} \), will be set to different values to compare the different results. After runs of the proposed hybrid intelligent algorithm (1000 uncertain random simulations, and 1000 generations in GA), it can be seen from Table 2 that
Table 2: Computational Results for the Model

<table>
<thead>
<tr>
<th>$\sigma_{pop}$</th>
<th>$\rho_{mut}$</th>
<th>$\rho_{crs}$</th>
<th>Cost</th>
<th>$\Delta_{best}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.6</td>
<td>0.4</td>
<td>40406</td>
<td>0.005</td>
</tr>
<tr>
<td>50</td>
<td>0.5</td>
<td>0.4</td>
<td>40416</td>
<td>0.030</td>
</tr>
<tr>
<td>50</td>
<td>0.5</td>
<td>0.7</td>
<td>40455</td>
<td>0.126</td>
</tr>
<tr>
<td>60</td>
<td>0.4</td>
<td>0.5</td>
<td>40404</td>
<td>0.000</td>
</tr>
<tr>
<td>60</td>
<td>0.3</td>
<td>0.4</td>
<td>40447</td>
<td>0.106</td>
</tr>
<tr>
<td>60</td>
<td>0.6</td>
<td>0.7</td>
<td>40465</td>
<td>0.151</td>
</tr>
<tr>
<td>70</td>
<td>0.6</td>
<td>0.7</td>
<td>40420</td>
<td>0.040</td>
</tr>
<tr>
<td>70</td>
<td>0.8</td>
<td>0.5</td>
<td>40441</td>
<td>0.092</td>
</tr>
<tr>
<td>70</td>
<td>0.4</td>
<td>0.4</td>
<td>40457</td>
<td>0.131</td>
</tr>
</tbody>
</table>

the “$\Delta_{best}$”s, calculated by the formula: (actual value - best value)/ best value × 100%, do not exceed 0.151%, which does not exceed the general project demand. Note that the “best value” here means the minimal value among the costs in Table 2.

6 Conclusion

In this paper, we introduced uncertainty theory and uncertain random programming into project scheduling problem for subscribing the environment with co-existed randomness and uncertainty. The uncertain random expected cost minimization model was built. The uncertain random project scheduling model can be degenerated to random model and uncertain model, respectively. Hence, the proposed model can be considered as the extension of PSP model in single uncertain or random environment. For some special case, the proposed uncertain random programming model was transformed to a crisp mathematical programming model. Besides, a hybrid intelligent algorithm integrating uncertain random simulations and GA was also introduced for searching the quasi-optimal schedule.

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References


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