Uncertain pricing problem in supply chain with a common dominant retailer

Hu Huang and Hua Ke*

School of Economics and Management, Tongji University, Shanghai 200092, China

Abstract

In this paper, we consider a pricing problem in a supply chain with two different manufacturers which distribute differentiated but substitutable products through a common dominant retailer. The manufacturing costs, the sale costs and the demands are characterized by uncertain variables. Meanwhile, uncertainty theory and Stackelberg Nash modeling method are employed to formulate the pricing problem. Their optimal pricing decisions on wholesale prices and retailer markups are derived from the proposed Stackelberg Nash model. Numerical experiments are also given to demonstrate the effectiveness of the uncertain model.

Keywords: Pricing decision, Uncertainty theory, Stackelberg Nash model, Supply chain, Super retailer

1. Introduction

The power of manufacturers and retailers has reversed since many super retailers like Walmart, Carrefour appeared. Thus, considerable attentions from scholars and practitioners have been focused on the dominant downstream members. In this paper, we investigate a pricing problem in some widely existing competing supply chains, in which differentiated but substitutable products are manufactured and sold into the same market. These chains often include many

*Corresponding author.
Email: 13huanghu@tongji.edu.cn (Hu Huang) hke@tongji.edu.cn (Hua Ke)
upstream manufacturers but only few super downstream retailers, and each retailer distributes multiple substitutable products from various manufacturers. Choi (1991, 1996) initiated the research of the pricing problems with two manufacturers which sell products through one common retailer. It is found that with a linear demand function, a manufacturer is better off by maintaining exclusive dealers while a retailer has an incentive to deal with several producers. However, both of them typically focus on deterministic demands and costs. In fact, the real world exists many nondeterministic factors which cannot be ignored when making pricing decisions. Those nondeterministic factors, for instance material costs, customer incomes, workers’ expenses and technology improvement, usually affect manufacturing costs or consumers’ demands (Wu et al., 2009; Shi et al., 2013).

However, if accurate distributions cannot be estimated due to the lack of historical data not only for economic reasons, but also for technical difficulties, then the market-bases or consumers’ demands can usually be determined by experts due to the complicated and changeable environments. Thus, fuzzy set theory can be a very powerful tool to handle complicated nondeterministic environment embedded with these vague factors. In fact, many scholars have introduced the concept of fuzzy set theory, initiated by Zadeh (1965), to pricing decision problems recently (Ryu and Yücesan, 2010; Zhao et al., 2012a,b).

When indeterminate phenomena behave neither randomness nor fuzziness, uncertainty theory, initiated by Liu (2007) and refined by Liu (2010) can be a better choice. Uncertainty theory is a branch of axiomatic mathematics for modeling human uncertainty, which has been successfully employed for explaining many uncertain decision-making situations, e.g., facility location (Gao, 2012), portfolio selection (Bhattacharyya et al., 2013), differential games (Yang and Gao, 2013), assignment problem (Zhang and Peng, 2013), inventory problem (Ding, 2014), network problem (Han et al., 2014) and time-cost trade-off problem (Ke, 2014).

Thus far, to the best of our knowledge, there are no researches on pricing problem by uncertainty theory. This paper will fill the gap as mentioned. Specif-
ically, the manufacturing costs and the demands are characterized as uncertain variables, decided by domain experts or managers. Meanwhile, uncertainty theory and game-theory-based models are employed to formulate the pricing decision problem. Our main interest is to investigate how the two manufacturers and the common retailer make their own pricing decisions on wholesale prices and retail markups when facing uncertain environment.

The reminder of this paper is as follows: In Section 2, we present a brief introduction of uncertainty theory as well as the expected uncertain programming model. In Section 3, a Stackelberg Nash model based on uncertainty theory is employed to model the pricing decision problem. Following that, the solution approach is also presented. In Section 4, numerical experiments are applied to demonstrate the effectiveness of the model. Some conclusions are drawn in Section 5.

2. Preliminaries

In this section, we will introduce some important concepts and theorems about uncertain variable for modeling the pricing decision problem with uncertain factors.

Let Γ be a nonempty set, and L a σ-algebra over Γ. Each element Λ in L is called an event. Uncertain measure is defined as a function from L to [0, 1]. In detail, the concept of uncertain measure is pioneered by Liu (2007) and refined by Liu (2010). Uncertain measure M is a set function defined over the following four axioms:

**Axiom 1.** *(Normality Axiom)* $M(\Gamma) = 1$.

**Axiom 2.** *(Duality Axiom)* $M(\Lambda) + M(\Lambda^c) = 1$.

**Axiom 3.** *(Subadditivity Axiom)* For every countable sequence of events $\{\Lambda_i\}$, $i = 1, 2, ..., we have

$$M(\bigcup_{i=1}^{\infty} \Lambda_i) \leq \sum_{i=1}^{\infty} M(\Lambda_i).$$
Axiom 4. (Product Measure Axiom) Let $(\Gamma_k, L_k, M_k)$ be uncertainty spaces for $k = 1, 2, \cdots$. The product uncertain measure $M$ is an uncertain measure satisfying

$$M \left( \prod_{k=1}^{\infty} A_k \right) = \bigwedge_{k=1}^{\infty} M_k \{A_k\}$$

where $A_k$ are arbitrarily chosen events from $L_k$ for $k = 1, 2, \cdots$, respectively.

Definition 1. (Liu, 2007) An uncertain variable is a measurable function $\xi$ from an uncertainty space $(\Gamma, L, M)$ to the set of real numbers, i.e., for any Borel set $B$ of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\}$$

is an event.

Sometimes, we should know the uncertainty distribution to model real-life uncertain optimization problems.

Definition 2. (Liu, 2007) The uncertainty distribution $\Phi$ of an uncertain variable $\xi$ is defined by

$$\Phi(x) = M\{\xi \leq x\}$$

for any real number $x$.

An uncertainty distribution $\Phi$ is said to be regular if its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in [0, 1]$.

Definition 3. (Liu, 2007) Let $\xi$ be an uncertain variable. The expected value of $\xi$ is defined by

$$E[\xi] = \int_{0}^{+\infty} M\{\xi \geq r\} dr - \int_{-\infty}^{0} M\{\xi \leq r\} dr$$

provided that at least one of the above two integrals is finite.

Lemma 1. (Liu, 2010) Let $\xi$ be an uncertain variable with uncertainty distribution $\Phi$. If the expected value exists, then

$$E[\xi] = \int_{0}^{+\infty} (1 - \Phi(x)) dx - \int_{-\infty}^{0} \Phi(x) dx.$$
Lemma 2. (Liu, 2010) Let $\xi$ be an uncertain variable with regular uncertainty distribution $\Phi$. If the expected value exists, then

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha)d\alpha.$$  

Example 1. The expected value of the zigzag uncertain variable $\xi = Z(a,b,c)$ is

$$E[\xi] = \int_0^{0.5} ((1 - 2\alpha)a + 2\alpha b)d\alpha + \int_{0.5}^1 ((2 - 2\alpha)b + (1 - 2\alpha)c)d\alpha$$

$$= \frac{a + 2b + c}{4}. \quad (1)$$  

Lemma 3. (Liu and Ha, 2010) Let $\xi_1, \xi_2, \cdots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. An uncertain function $f(x_1, x_2, \cdots, x_n)$ is strictly increasing with respect to $x_1, x_2, \cdots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \cdots, x_n$. Then the expected value of $\xi$ is $$f(\xi_1, \xi_2, \cdots, \xi_n)$$ is

$$E[\xi] = \int_0^1 f(\Phi_1^{-1}(\alpha), \cdots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \cdots, \Phi_n^{-1}(1-\alpha))d\alpha \quad (2)$$

provided that the expected value $E[\xi]$ exists.

Example 2. Let $\xi$ and $\eta$ be two positive independent uncertain variables with regular uncertainty distributions $\Phi$ and $\Psi$, respectively. Then we have

$$E[\xi] = \int_0^1 \frac{\Phi^{-1}(\alpha)}{\Psi^{-1}(1-\alpha)}d\alpha. \quad (3)$$

With the above concepts, we can model the pricing decision problem in uncertain environment.

3. Models and solution approaches

We restrict our research on two manufacturers which distribute differentiated but competing products through a common retailer per market area. The manufacturer (M_1 or M_2) produces product $i$ at a unit cost $c_i, i = 1, 2$ and wholesales the product to a common retailer $R$. Then the retailer retails the product to consumers with a unit sale cost $s_i$. The pricing decision problem
(how to set the wholesale prices for the manufacturers and how to choose a retail markup for the retailer) will be discussed in this paper.

In order to derive analytical solutions, just as many scholars do, linear demand function is assumed (Anderson and Bao, 2010):

$$Q_i = d_i - (\delta + \gamma)p_i + \gamma p_{3-i}, \quad i = 1, 2$$

where $d_i$ is the market base (the potential market size with all the prices equaling 0), changes in $d_i$ alter the relative product preferences, $\delta$ denotes the price-sensitive of the consumer, $\gamma$ is the substitutability of the two products, and $p_i = w_i + c_i$ is the sale price of product $i$. Let $\beta = \delta + \gamma$. This function can be rewritten as

$$Q_i = d_i - \beta p_i + \gamma p_{3-i}.$$  

All the uncertain coefficients are assumed nonnegative and mutually independent. The two manufacturers and the common retailer have full information on the demands and the costs of other channel members. We assume that the manufacturers and the retailer are risk neutral and willing to maximize the expected profit.

Moreover, we assume that the powerful retailer moves first and performs as Stackelberg leader. The decision sequence is as follows: The retailer can set the retail markups $(r_1, r_2)$ first and then the two manufacturers respectively choose their wholesale prices simultaneously. Following that, the retail prices are determined as $p_i = w_i + r_i, \quad i = 1, 2$, and the order quantities are also
If the conditions are the inverse functions, uncertainty distributions \( \Phi \), where

\[
\begin{align*}
\text{Proposition 1.} & \quad \text{uncertain function into an equivalent one first.}
\end{align*}
\]

To solve this Nash-Stackelberg game model, we should firstly transform the uncertain function into an equivalent one first.

\[
\begin{align*}
\max_{r_1, r_2} \pi_r &= \sum_{i=1}^{2} (r_i - \hat{s}_i)(\hat{d}_i - \beta(r_i + w_i^*) + \gamma(r_{3-i} + w_{3-i}^*))^+ \\
\text{subject to:} & \quad \mathcal{M}\{w_1 - \hat{s}_1 \leq 0\} = 0 \\
& \quad \mathcal{M}\{w_2 - \hat{s}_2 \leq 0\} = 0
\end{align*}
\]

where \((w_1^*, w_2^*)\) solves problems:

\[
\begin{align*}
\max_{w_1} \pi_{m_1} &= (w_1 - \hat{c}_1)(\hat{d}_1 - \beta(r_1 + w_1) + \gamma(r_2^* + w_2))^+ \\
\max_{w_2} \pi_{m_2} &= (w_2 - \hat{c}_2)(\hat{d}_2 - \beta(r_2 + w_2) + \gamma(r_1^* + w_1))^+ \\
\text{subject to:} & \quad \mathcal{M}\{\hat{d}_i - \beta(r_1 + w_1) + \gamma(r_{3-i} + w_{3-i}) \leq 0\} = 0 \\
& \quad \mathcal{M}\{w_i - \hat{s}_i \leq 0\} = 0, \quad i = 1, 2.
\end{align*}
\]

To solve this Nash-Stackelberg game model, we should firstly transform the uncertain function into an equivalent one first.

\[
\begin{align*}
E[\pi_{m_1}] &= -\beta w_1^2 + \gamma w_2 w_1 + (-\beta r_1 + \gamma r_2 + E[(\hat{d}_1) + \beta E[\hat{c}_1]]w_1 + E(\hat{c}_1)\beta r_1 \\
& \quad - \gamma(r_2 + w_2)E(\hat{c}_1) - \int_0^1 [\Phi_{c_1}^{-1}(1 - \alpha)\Phi_{d_1}^{-1}(\alpha)]d\alpha \\
E[\pi_{m_2}] &= -\beta w_2^2 + \gamma w_1 w_2 + (-\beta r_2 + \gamma r_1 + E[(\hat{d}_2) + \beta E[\hat{c}_2]]w_2 + \beta r_2 E(\hat{c}_2) \\
& \quad - \gamma(r_1 + w_1)E(\hat{c}_2) - \int_0^1 [\Phi_{c_2}^{-1}(1 - \alpha)\Phi_{d_2}^{-1}(\alpha)]d\alpha \\
E[\pi_r] &= \sum_{i=1}^{2} \{-\beta r_i^2 + \gamma r_{3-i} r_i + (-\beta w_i + \gamma w_{3-i} + E[(\hat{d}_i) + \beta E[\hat{s}_i]]r_i + E(\hat{s}_i)\beta w_i \\
& \quad - \gamma(r_{3-i} + w_{3-i})E(\hat{s}_i) - \int_0^1 [\Phi_{s_i}^{-1}(1 - \alpha)\Phi_{d_i}^{-1}(\alpha)]d\alpha}\}
\end{align*}
\]

where \(\hat{c}_i, \hat{s}_i\) and \(\hat{d}_i\) are positive independent uncertain variables with the regular uncertainty distributions \(\Phi_{c_i}, \Phi_{s_i}\), and \(\Phi_{d_i}\), and \(\Phi_{c_i}^{-1}(\alpha), \Phi_{s_i}^{-1}(\alpha)\) and \(\Phi_{d_i}^{-1}(\alpha)\) are the inverse functions, \(i = 1, 2, \alpha \in [0, 1]\), respectively.
Remark 1. All the propositions are conditional on the assumptions that all the constraints in the model are satisfied. In addition, the proofs of the propositions are provided in appendix.

Then, we should derive the Nash equilibrium in the lower level for the given markup policies \( r_1 \) and \( r_2 \) specified by the retailer.

Proposition 2. For the given markup policies \( r_1 \) and \( r_2 \), which are specified by the common retailer in advance, the two manufacturers’ optimal wholesale prices are:

\[
\begin{align*}
    w_1^*(r_1, r_2) &= \frac{(-2\beta^2 + \gamma^2) r_1 + \beta \gamma r_2 + 2\beta (E[\tilde{d}_1] + \beta E[\tilde{c}_1]) + \gamma (E[\tilde{d}_2] + \beta E[\tilde{c}_2])}{4\beta^2 - \gamma^2}, \\
    w_2^*(r_1, r_2) &= \frac{(-2\beta^2 + \gamma^2) r_2 + \beta \gamma r_1 + 2\beta (E[\tilde{d}_2] + \beta E[\tilde{c}_2]) + \gamma (E[\tilde{d}_1] + \beta E[\tilde{c}_1])}{4\beta^2 - \gamma^2}.
\end{align*}
\]

The common retailer, based on the followers’ responses, selects the most profitable markup policies for itself.

Proposition 3. Given the Nash equilibrium in the lower level, the equilibrium markup prices are:

\[
\begin{align*}
    w_1^* &= \frac{A(C_1 + E[\tilde{d}_1]) + (A^2 - B^2) E[\tilde{s}_1] + B(C_2 + E[\tilde{d}_2])}{2(A^2 - B^2)}, \\
    w_2^* &= \frac{A(C_2 + E[\tilde{d}_2]) + (A^2 - B^2) E[\tilde{s}_2] + B(C_1 + E[\tilde{d}_1])}{2(A^2 - B^2)}
\end{align*}
\]

where

\[
\begin{align*}
    A &= \frac{2\beta^3 - \beta \gamma^2}{4\beta^2 - \gamma^2}, \\
    B &= \frac{\beta^2 \gamma}{4\beta^2 - \gamma^2}, \\
    C_1 &= \frac{(-2\beta^2 + \gamma^2)(E[\tilde{d}_1] + \beta E[\tilde{c}_1]) + \beta \gamma ((E[\tilde{d}_2] + \beta E[\tilde{c}_2]))}{4\beta^2 - \gamma^2}, \\
    C_2 &= \frac{(-2\beta^2 + \gamma^2)(E[\tilde{d}_2] + \beta E[\tilde{c}_2]) + \beta \gamma ((E[\tilde{d}_1] + \beta E[\tilde{c}_1]))}{4\beta^2 - \gamma^2}.
\end{align*}
\]

4. Numerical examples

In this section, numerical examples are provided to explore the effectiveness of the proposed uncertain model.
Consider the case in which no dominant power or technology advantage exists among the two supply manufacturers. Due to the lack of historical data or the changeable environment, the costs and market bases of the two products are predicted by experienced specialists. The linguist expressions of the experts and the corresponding uncertain variables are presented in Table 1. We assume that $\beta = 200$, and $\gamma$ varies from $100 \rightarrow 180$, that’s to say, the competing intensity $\theta = \frac{\gamma}{\beta} \in [0.5, 0.9]$. The optimal decisions with different competing intensity are derived and presented in Table 2.

Table 1: Uncertain variables

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Linguist description</th>
<th>Distribution</th>
<th>Expected value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{c}_1, \tilde{c}_2$</td>
<td>between 5 ~ 7</td>
<td>$L(5,7)$</td>
<td>6</td>
</tr>
<tr>
<td>$\tilde{s}_1, \tilde{s}_2$</td>
<td>between 3 ~ 5</td>
<td>$L(3,5)$</td>
<td>4</td>
</tr>
<tr>
<td>$\tilde{d}_1, \tilde{d}_2$</td>
<td>about 3500</td>
<td>$Z(3000,3500,4000)$</td>
<td>3500</td>
</tr>
</tbody>
</table>

Table 2: The expected profits of the participants

<table>
<thead>
<tr>
<th>$\text{Int}$</th>
<th>$w_1^*$</th>
<th>$E[\pi_{m1}]$</th>
<th>$w_2^*$</th>
<th>$E[\pi_{m2}]$</th>
<th>$(r_1^<em>, r_2^</em>)$</th>
<th>$E[\pi_r]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>10.8333</td>
<td>15255.56</td>
<td>10.8333</td>
<td>15255.56</td>
<td>(14.5000,14.5000)</td>
<td>28033.33</td>
</tr>
<tr>
<td>0.6</td>
<td>11.3929</td>
<td>16399.91</td>
<td>11.3929</td>
<td>16399.91</td>
<td>(18.8750,18.8750)</td>
<td>40716.07</td>
</tr>
<tr>
<td>0.7</td>
<td>12.0385</td>
<td>17875.94</td>
<td>12.0385</td>
<td>17875.94</td>
<td>(26.1667,26.167)</td>
<td>63202.56</td>
</tr>
<tr>
<td>0.8</td>
<td>12.7917</td>
<td>19808.68</td>
<td>12.7917</td>
<td>19808.68</td>
<td>(40.7500,40.7500)</td>
<td>110704.17</td>
</tr>
<tr>
<td>0.9</td>
<td>13.6818</td>
<td>22385.40</td>
<td>13.6818</td>
<td>22385.40</td>
<td>(84.5000,84.5000)</td>
<td>259645.45</td>
</tr>
</tbody>
</table>

5. Conclusion

In this paper, we investigate the supply chain pricing problem in which two manufacturers compete to sell differentiated but substitutable products into the same market through a common retailer. The manufacturing costs, sales costs and the demands are characterized by uncertain variables, which are more in line with the real-life situations. Meanwhile, uncertainty theory and Stackelberg Nash modeling approach have been employed to formulate the pricing decision.
problem. How to make their own pricing decisions on wholesale prices and retailer markups in face of other participants are derived from the proposed model. Numerical experiments demonstrate that the proposed model is effective.

Acknowledgments

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6. Appendix

6.1. Proof of Proposition 1

Let \( \tilde{c}_i, \tilde{s}_i \) and \( \tilde{d}_i \) be positive independent uncertain variables with the regular uncertainty distributions \( \Phi_{c_i}, \Phi_{s_i}, \) and \( \Phi_{d_i}. \) Obviously, \( E[\pi_{m1}] \) is monotone increasing with \( \tilde{d}_1 \) and monotone decreasing with \( \tilde{c}_1 \). Then referring to Lemma 2, we have:

\[
E[\pi_{m1}] = \int_0^1 [(w_1 - \Phi^{-1}_{c_i}(1 - \alpha))(\Phi^{-1}_{d_i}(\alpha) - \beta(r_1 + w_1) + \gamma(r_2 + w_2))] d\alpha = w_1 E[\tilde{d}_1] - \beta(r_1 + w_1)w_1 + \gamma(r_2 + w_2)w_1 + \beta(r_1 + w_1)E[\tilde{c}_1] - \gamma(r_2 + w_2)E[\tilde{c}_1] - \int_0^1 [\Phi^{-1}_{c_i}(1 - \alpha)\Phi^{-1}_{d_i}(\alpha)] d\alpha
\]

\[
= -\beta w_1^2 + \gamma w_2 w_1 + (\beta r_1 + \gamma r_2 + E[\tilde{d}_1] + \beta E[\tilde{c}_1])w_1 + E(\tilde{c}_1)\beta r_1 - \gamma(r_2 + w_2)E(\tilde{c}_1) - \int_0^1 [\Phi^{-1}_{c_i}(1 - \alpha)\Phi^{-1}_{d_i}(\alpha)] d\alpha.
\]

In the same way, we can get the crisp expected profit function of \( E[\pi_{m2}] \) and \( E[\pi_r] \) easily.

\[
E[\pi_{m2}] = -\beta w_2^2 + \gamma w_1 w_2 + (\beta r_2 + \gamma r_1 + E[\tilde{d}_2] + \beta E[\tilde{c}_2])w_2 + E(\tilde{c}_2)\beta r_2 - \gamma(r_1 + w_1)E(\tilde{c}_2) - \int_0^1 [\Phi^{-1}_{c_2}(1 - \alpha)\Phi^{-1}_{d_2}(\alpha)] d\alpha.
\]

\[
E[\pi_{m2}] = \sum_{i=1}^{i} [(-\beta r_i^2 + \gamma r_{3-i} r_i + (\beta w_i + \gamma w_{3-i} + E[\tilde{d}_i] + \beta E[\tilde{s}_i])r_i + E(\tilde{s}_i)\beta w_i - \gamma(r_{3-i} + w_{3-i})E(\tilde{s}_i) - \int_0^1 [\Phi^{-1}_{s_i}(1 - \alpha)\Phi^{-1}_{d_i}(\alpha)] d\alpha.
\]

10
Proposition 1 is proved.

6.2. Proof of Proposition 2

Referring to the crisp expected profit functions of $E[\pi_m]$ and $E[\pi_m]$ in Proposition 1, we can obtain the second-order derivatives of the objective functions $\pi_{m_1}(w_1, w_2)$ and $\pi_{m_2}(w_1, w_2)$ as follows:

\[
\begin{align*}
\frac{\partial^2 \pi_{m_1}(w_1, w_2)}{\partial w_1^2} &= -2\beta; \quad \frac{\partial^2 \pi_{m_1}(w_1, w_2)}{\partial w_1 \partial w_2} = \gamma; \\
\frac{\partial^2 \pi_{m_2}(w_1, w_2)}{\partial w_2^2} &= -2\beta; \quad \frac{\partial^2 \pi_{m_2}(w_1, w_2)}{\partial w_2 \partial w_1} = \gamma.
\end{align*}
\]

Then the Hessian matrix can be attained:

\[
H_1 = \begin{bmatrix}
\frac{\partial^2 \pi_{m_1}(w_1, w_2)}{\partial w_1^2} & \frac{\partial^2 \pi_{m_1}(w_1, w_2)}{\partial w_1 \partial w_2} \\
\frac{\partial^2 \pi_{m_2}(w_1, w_2)}{\partial w_2^2} & \frac{\partial^2 \pi_{m_2}(w_1, w_2)}{\partial w_2 \partial w_1}
\end{bmatrix} = \begin{bmatrix}
-2\beta & \gamma \\
\gamma & -2\beta
\end{bmatrix}
\]

where $H_1$ is a negative definite matrix with the assumptions that $\beta > 0$ and $\beta > \gamma$. Hence $\pi_{m_1}(w_1, w_2)$ and $\pi_{m_2}(w_1, w_2)$ are jointly concave in $w_1, w_2$.

Therefore we can get the Nash equilibrium by setting the first-order derivatives:

\[
\begin{align*}
\frac{\partial E[\pi_{m_1}]}{\partial w_1} &= -2\beta w_1 + \gamma w_2 - \beta r_1 + \gamma r_2 + E[\tilde{d}_1] + \beta E[\tilde{c}_1] = 0, \\
\frac{\partial E[\pi_{m_2}]}{\partial w_2} &= -2\beta w_2 + \gamma w_1 - \beta r_2 + \gamma r_1 + E[\tilde{d}_2] + \beta E[\tilde{c}_2] = 0.
\end{align*}
\]

The followers’ optimal responses to $(r_1, r_2)$ can be easily obtained by solving the following two equations:

\[
\begin{align*}
w_1^*(r_1, r_2) &= \frac{(-2\beta^2 + \gamma^2)r_1 + \beta \gamma r_2 + 2\beta (E[\tilde{d}_1] + \beta E[\tilde{c}_1]) + \gamma (E[\tilde{d}_2] + \beta E[\tilde{c}_2])}{4\beta^2 - \gamma^2}, \\
w_2^*(r_1, r_2) &= \frac{(-2\beta^2 + \gamma^2)r_2 + \beta \gamma r_1 + 2\beta (E[\tilde{d}_2] + \beta E[\tilde{c}_2]) + \gamma (E[\tilde{d}_1] + \beta E[\tilde{c}_1])}{4\beta^2 - \gamma^2}.
\end{align*}
\]

Proposition 2 is proved.
6.3. Proof of Proposition 3

Substituting Eqn. 5 into the profit function of the retailer, we obtain:

\[
\begin{align*}
E[\pi_r] &= \sum_{i=1}^{2} E[(r_i - \hat{s}_i)(\hat{d}_i - Ar_i + Br_{3-i} + C_i) + \\
&= 2\{E[\hat{d}_i]r_i - Ar_i^2 + Br_{3-i}r_i + C_ir_i \\
&- \int_{0}^{1} [\Phi_{s_i}^{-1}(1 - a)\Phi_{d_i}^{-1}(a)]da \\
&+ E[\hat{s}_i]Ar_i - E[\hat{s}_i]Br_{3-i} - E[\hat{s}_i]C_i \}
\end{align*}
\]

(9)

where

\[
A = \frac{2\beta^3 - \beta \gamma^2}{4\beta^2 - \gamma^2}, \quad B = \frac{\beta^2 \gamma}{4\beta^2 - \gamma^2},
\]

\[
C_1 = \frac{(-2\beta^2 + \gamma^2)(E(\hat{d}_1) + \beta E(\hat{c}_1)) + \beta \gamma((E(\hat{d}_2) + \beta E(\hat{c}_2)))}{4\beta^2 - \gamma^2},
\]

\[
C_2 = \frac{(-2\beta^2 + \gamma^2)(E(\hat{d}_2) + \beta E(\hat{c}_2)) + \beta \gamma((E(\hat{d}_1) + \beta E(\hat{c}_1)))}{4\beta^2 - \gamma^2}.
\]

Referring to Eqn. 9, we can get the second-order derivative of the equivalent objective function \(\pi_r(r_1, r_2)\) as follows:

\[
\begin{align*}
\frac{\partial^2 \pi_r(r_1, r_2)}{\partial r_1^2} &= -2\beta(2\beta^2 - \gamma^2), \quad \frac{\partial^2 \pi_r(r_1, r_2)}{\partial r_1 \partial r_2} = \frac{\beta^2 \gamma}{4\beta^2 - \gamma^2}; \\
\frac{\partial^2 \pi_r(r_1, r_2)}{\partial r_2^2} &= -\frac{2\beta}{4\beta^2 - \gamma^2}, \quad \frac{\partial^2 \pi_r(r_1, r_2)}{\partial r_2 \partial r_1} = \frac{\beta^2 \gamma}{4\beta^2 - \gamma^2}.
\end{align*}
\]

The Hessian matrix is

\[
H_2 = \begin{vmatrix}
\frac{\partial^2 \Pi_r(p_1, p_2)}{\partial p_1^2} & \frac{\partial^2 \Pi_r(p_1, p_2)}{\partial p_1 \partial p_2} \\
\frac{\partial^2 \Pi_r(p_1, p_2)}{\partial p_2 \partial p_1} & \frac{\partial^2 \Pi_r(p_1, p_2)}{\partial p_2^2}
\end{vmatrix}
\]

\[
= \begin{vmatrix}
\frac{-2\beta(2\beta^2 - \gamma^2)}{4\beta^2 - \gamma^2} & \frac{\beta^2 \gamma}{4\beta^2 - \gamma^2} \\
\frac{\beta^2 \gamma}{4\beta^2 - \gamma^2} & \frac{-2\beta(2\beta^2 - \gamma^2)}{4\beta^2 - \gamma^2}
\end{vmatrix}
\]

With the assumption that \(\beta > \gamma > 0\), we get

\[
\frac{2\beta(2\beta^2 - \gamma^2)}{4\beta^2 - \gamma^2} > \frac{2\beta^3}{4\beta^2 - \gamma^2} > \frac{\beta^2 \gamma}{4\beta^2 - \gamma^2} > 0.
\]

Hence \(H_2\) is a negative definite matrix and \(E[\pi_r(r_1, r_2)]\) is jointly concave in \(r_1, r_2\). Therefore, we can get the equilibrium markup prices by setting the
The leaders’ equilibrium prices \((r_1, r_2)\) can be easily obtained by solving the two equations:

\[
\begin{align*}
\frac{\partial \pi_1(r_1, r_2)}{\partial r_1} &= E(\tilde{d}_1) - 2A r_1 + 2B r_2 + C_1 + AE[\tilde{s}_1] - BE[\tilde{s}_2] = 0 \\
\frac{\partial \pi_1(r_1, r_2)}{\partial r_1} &= E(\tilde{d}_2) - 2A r_2 + 2B r_1 + C_2 + E(\tilde{s}_2)A - BE[\tilde{s}_1] = 0.
\end{align*}
\]

The leaders’ equilibrium prices \((r_1, r_2)\) can be easily obtained by solving the two equations:

\[
\begin{align*}
\frac{\partial \pi_1}{\partial r_1} &= A(C_1 + E[\tilde{d}_1]) + (A^2 - B^2)E[\tilde{s}_1] + B(C_2 + E[\tilde{d}_2]), \\
\frac{\partial \pi_1}{\partial r_2} &= A(C_2 + E[\tilde{d}_2]) + (A^2 - B^2)E[\tilde{s}_2] + B(C_1 + E[\tilde{d}_1]),
\end{align*}
\]

where

\[
\begin{align*}
A &= \frac{2\beta^3 - \beta \gamma^2}{4\beta^2 - \gamma^2}, & B &= \frac{\beta \gamma}{4\beta^2 - \gamma^2}, \\
C_1 &= \frac{(-2\beta^2 + \gamma^2)(E(\tilde{d}_1) + \beta E(\tilde{c}_1)) + \beta \gamma((E(\tilde{d}_2) + \beta E(\tilde{c}_2)))}{4\beta^2 - \gamma^2}, \\
C_2 &= \frac{(-2\beta^2 + \gamma^2)(E(\tilde{d}_2) + \beta E(\tilde{c}_2)) + \beta \gamma((E(\tilde{d}_1) + \beta E(\tilde{c}_1)))}{4\beta^2 - \gamma^2}.
\end{align*}
\]

**Proposition 3** is proved.

### References


