Pricing decision problem in uncertain supply chain with dual distribution channels

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Abstract

Recently, as online shopping has become an important distribution channel, most manufacturers distribute their products through both traditional channels (referred to as “r-channel”) and online channels (referred to as “e-channel”). In this paper, we consider a pricing decision problem in supply chains in which the manufacturer sells to a retailer as well as to consumers through a company store (e.g., online shop or direct-sale store) directly. The demands are characterized as a price-dependent and channel-dependent (r-channel v.s. e-channel) uncertain functions. Meanwhile, manufacturing costs and sales costs are characterized as uncertain variables. In addition, uncertain expected bilevel models are employed to explore how the channel members make their optimal pricing decisions under different power structures. Numerical experiments are also conducted to examine the effects of the power structures and uncertain degrees on the equilibrium prices and expected profits of the participants. It is revealed that consumers will suffer from higher prices when facing uncertain environment. The supply chain members may benefit from higher uncertain degrees of their own costs, whereas the other channel member will gain less profit. Some more managerial highlights are presented in this paper.

Keywords: supply chain, pricing decision, dual channels, uncertain variable, Stackelberg game

1. Introduction

With the development of information technology and e-commerce, online shopping by now has become an important distribution channels. according to new figures from eMarketer (EMarketer, 2014), Business-to-Consumer (B2C) e-commerce sales worldwide will reach $1.471 trillion in 2014, increasing nearly 20% over 2013. Generally, considerable manufacturers including Apple, Nike and other brands have employed online channels to complement independent retailers’ channel such as Wal-Mart and Best Buy. In addition, manufacturers like Dell, who traditionally deliver their products to customers directly, are now going to use the dual channels integrating their direct channel (e-channel) and the retail channel (r-channel). In China, most of brands have opened its own brand store in Tmall (http://www.tmall.com), one of the biggest e-commerce platform in the world, to complement their traditional channel. By now, the hybrid channel has become one of the most common channel structure in many industries.

Thus, increasing studies from scholars and practitioners have been focused on dual channels. Webb (2002) found that the dual channels, especially the online channel, can potentially reduce costs and result in an increasing margin. Park and Keh (2003) focused on the pricing decision-making problem of the manufacturer and retailers in supply chains with hybrid channels. Chiang et al. (2003) showed that when an online channel opens, both manufacturer and retailer can reap benefits even if no sales occur in the online channel. Yao and Liu (2005) studied the pricing equilibria by using the Bertrand and Stackelberg competition models. Huang and Swaminathan (2009) concentrated on the optimal pricing strategies when a product is sold on two channels such as the Internet and a traditional channel and explored the behaviors (prices and profits) under different parameters and consumer preferences for the alternative channels. Chen et al. (2013) considered a pricing policies in supply chain with one manufacturer, who sells its products

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directly through internet channel and indirectly through a traditional retailer. Different from the literature above, they assumed that the retailer in the supply chain also sells a substitutable product manufactured by another manufacturer.

The works above have typically concentrated on deterministic demands and costs. The real world, however, exists many indeterministic factors which cannot be ignored when making pricing decisions. Those indeterministic factors, for instance, material costs, customer incomes, workers' expenses and technology improvements, usually make costs and demands ambiguous and unpredictable. Some of them can be described as random variables if their distributions can accurately be attained. Then, probability theory can be employed to deal with these random factors. In fact, Bernstein and Federgruen (2005) investigated the equilibrium behavior of decentralized supply chains with a single supplier servicing a network of competing retailers in random demand context. Hua et al. (2006) studied a two-echelon supply chain consisting of a manufacturer and a retailer for a single-period product and described the retail-market demand uncertainty by coefficient of variation. Dumrongsi et al. (2008) considered a price and service competition problem in dual channel supply chains in which a manufacturer sells to a retailer as well as to consumers directly, and found that demand variability plays a key role in the manufacturer's motivation for opening a direct channel. Shi et al. (2013) applied a game-theory-based framework to model power in a supply chain with random and price-dependent demand and examine how demand models affect supply chain participants' performance. Li et al. (2014) investigated a pricing problem in dual-channel supply chain with one risk-neutral manufacturer and one risk-averse retailer under stochastic demand.

Another type of indeterministic factors which has been well studied is fuzziness. By now, fuzzy set theory, proposed by Zadeh (1965), has been widely applied to this supply chain pricing problems, e.g., Liu and Xu (2014), Zhao et al. (2012), Wei and Zhao (2014), etc. Specially, Soleimani (2014) analyzed a pricing decision problem of a dual-channel supply chain in fuzzy environment. They assumed that manufacturer, who acts as a Stackelberg leader, produces a product and sells it to the end customers through a retailer or directly.

Nowadays, the products, especially hi-tech devices, are usually updated quickly. The demands and costs of these products are often with no historical data. However, knowing the distributions of the demands and costs is essential for the managers to make decisions, e.g. pricing the products, adding investment, determining the production quantities. In these cases, we have to rely on belief degrees given by experienced managers and experts to estimate the demands and costs. Nevertheless, surveys have indicated that human beings usually estimate a much wider range of values than it actually takes. Therefore, human belief degree should not be treated as random variable or fuzzy variable. When some of the indeterminate phenomena behave neither randomness nor fuzziness, uncertainty theory, initiated by Liu (2007) and refined by Liu (2010), can be a useful tool. By now, the new theory has been well developed and applied in a wide variety of real problems, e.g., facility location (Gao, 2012), stock problem (Chen, 2012), differential games (Yang and Gao, 2013), inventory problem (Ding, 2013), project scheduling problem (Ke, 2014) and assignment problem (Zhang and Peng, 2013). Recently, Huang and Ke (2014) applied the uncertainty theory to a pricing decision problem in supply chains with common retailer, but did not consider the pricing problem in supply chains with dual channels.

In this paper, uncertainty theory and game theory are employed to formulate the pricing decision problem in a supply chains consisting one retailer and one manufacturer. The manufacturer produces a product and sells it to the end customer through both traditional r-channel and e-channel directly. Consumers can choose the purchase channels for items based on prices and their preferences of the alternative channels. Specifically, the manufacturing costs and sales costs are characterized as uncertain variables, whose distributions are estimated by experts or managers. Our main interest is to examine how the manufacturer and the retailer make their own pricing decisions on wholesale prices and retail markups when facing uncertain environment under different power structures. Thus, uncertainty-theory and game-theory-based models are employed to formulate the pricing decision problem and derive the optimal equilibria. Numerical experiments are also given to explore the impacts of power structures and uncertain degrees on the equilibrium prices and expected profits of the participants.

This paper has three contributions to the extant literature. Firstly, we employ uncertainty theory to formulate managers' or experts' estimations about the product demands and costs which are often of no history data. How should the dual-channel supply chain members make their own optimal pricing decisions are derived through the uncertainty-theory-based and game-theory-based models. Secondly, we consider three possible power structures (manufacturer-dominant, retailer-dominant and non-dominant), while in most extant literature (Soleimani, 2014), only manufacturer-dominant cases were researched. Thirdly, we mainly analyse the effects of the power structures and parameters' uncertainty degrees on the pricing decisions and performance of the supply chain. Some interesting managerial
highlights are illustrated in this paper.

The rest of this paper is as follows. Initially, some useful concepts and theorems of uncertainty theory are presented in Section 2. Section 3 discusses some necessary assumptions of the problem and provides the crisp form of the expected profits functions. In Section 4, two Stackelberg and a Nash game models based on uncertainty theory are employed to derive the equilibrium pricing decisions of the participants in different power structures. In Section 5, numerical experiments are applied to demonstrate the effectiveness of the models as well as to examine the impacts of the power structures and parameters’ uncertain degrees on the prices and the expected profits of the participants. Some conclusions are drawn in Section 6.

2. Preliminaries

In this section, we will recall some important concepts and theorems of uncertainty theory, which is a branch of axiomatic mathematics for modeling human uncertainty, for modeling the pricing decision problem with uncertain factors.

Let $\Gamma$ be a nonempty set, $\mathcal{L}$ a $\sigma$-algebra over $\Gamma$, and each element $\Lambda$ in $\mathcal{L}$ is called an event. Uncertain measure $M$ is defined as a function from $\mathcal{L}$ to $[0, 1]$.

**Definition 1.** (Liu, 2007) The set function $M$ is called an uncertain measure if it satisfies:

**Axiom 1.** (Normality Axiom). $M(\Gamma) = 1$ for the universal set $\Gamma$.

**Axiom 2.** (Duality Axiom). $M(\Lambda) + M(\Lambda^c) = 1$ for any event $\Lambda$.

**Axiom 3.** (Subadditivity Axiom). For every countable sequence of events $\Lambda_1, \Lambda_2, \cdots$, we have:

$$M\left(\bigcup_{i=1}^{\infty} \Lambda_i\right) \leq \sum_{i=1}^{\infty} M(\Lambda_i).$$

Besides, the product uncertain measure on the product $\sigma$-algebra $\mathcal{L}$ was defined by ? as follows:

**Axiom 4.** (Product Axiom). Let $(\Gamma_k, \mathcal{L}_k, M_k)$ be uncertainty spaces for $k = 1, 2, \cdots$. The product uncertain measure $M$ is an uncertain measure satisfying

$$M\left(\prod_{k=1}^{\infty} A_k\right) = \bigwedge_{k=1}^{\infty} M_k(A_k)$$

where $A_k$ are arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1, 2, \cdots$, respectively.

**Definition 2.** (Liu, 2007) An uncertain variable is a measurable function $\xi$ from an uncertainty space $(\Gamma, \mathcal{L}, M)$ to the set of real numbers, i.e., for any Borel set $B$ of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\}$$

is an event.

**Definition 3.** (Liu, 2009) The uncertain variables $\xi_1, \xi_2, \cdots, \xi_n$ are said to be independent if

$$M\left(\bigcap_{i=1}^{n} (\xi_i \in B_i)\right) = \prod_{i=1}^{n} M(\xi_i \in B_i)$$

for any Borel sets $B_1, B_2, \cdots, B_n$.

Sometimes, we should know uncertainty distribution to model real-life uncertain optimization problems.
Definition 4. (Liu 2007) The uncertainty distribution $\Phi$ of an uncertain variable $\xi$ is defined by

$$\Phi(x) = M(\xi \leq x)$$

for any real number $x$.

An uncertainty distribution $\Phi$ is said to be regular if its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in [0, 1]$.

Definition 5. (Liu 2007) Let $\xi$ be an uncertain variable. The expected value of $\xi$ is defined by

$$E[\xi] = \int_0^{+\infty} M(\xi \geq r)dr - \int_{-\infty}^0 M(\xi \leq r)dr$$

provided that at least one of the above two integrals is finite.

Lemma 1. (Liu 2010) Let $\xi$ be an uncertain variable with uncertainty distribution $\Phi$. If the expected value exists, then

$$E[\xi] = \int_0^1 (1 - \Phi(x))dx - \int_{-\infty}^0 \Phi(x)dx.$$  

Lemma 2. (Liu 2010) Let $\xi$ be an uncertain variable with regular uncertainty distribution $\Phi$. If the expected value exists, then

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha)d\alpha.$$  

Example 1. The uncertainty distribution of a linear uncertain variable $\xi = L(a, b)$ is as follows:

$$\Phi(x) = \begin{cases} 
0, & x < a \\
(x - a)/(b - a), & a \leq x \leq b \\
1, & x > b
\end{cases}$$  

And the inverse uncertainty distribution is $\Phi^{-1}(\alpha) = a + (b - a)\alpha$, thus, the expected value can be attained

$$E[\xi] = \int_0^1 (a + (b - a)\alpha)d\alpha = \frac{a + b}{2}.$$  

Example 2. The uncertainty distribution of a zigzag uncertain variable $\xi = Z(a, b, c)$ is

$$\Phi(x) = \begin{cases} 
0, & x < a; \\
(x - a)/2(b - a), & a \leq x \leq b \\
(x + c - 2b)/(2(c - b)), & b < x \leq c \\
1, & x > c.
\end{cases}$$  

and its inverse uncertainty distribution is

$$\Phi^{-1}(\alpha) = \begin{cases} 
(1 - 2\alpha)a + 2\alpha b, & \alpha < 0.5 \\
(2 - 2\alpha)b + (2\alpha - 1)c, & \alpha \geq 0.5.
\end{cases}$$  

Thus, its expected value is as follows:

$$E[\xi] = \int_0^{0.5} ((1 - 2\alpha)a + 2\alpha b)d\alpha + \int_{0.5}^1 ((2 - 2\alpha)b + (1 - 2\alpha)c)d\alpha = \frac{a + 2b + c}{4}.$$  

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Lemma 3. [Liu and Ha, 2010] Let $\xi_i$ be independent uncertain variables with regular uncertainty distributions $\Phi_i$, $i = 1, 2, \cdots, n$, respectively, and $f(x_1, x_2, \cdots, x_n)$ be strictly increasing with respect to $x_1, x_2, \cdots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \cdots, x_n$. Then the expected value of $\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$ can be defined by

$$E[\xi] = \int_0^1 (\Phi_1^{-1}(\alpha), \cdots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \cdots, \Phi_n^{-1}(1-\alpha))d\alpha$$

(6)

provided that the expected value $E[\xi]$ exists.

Example 3. Let $\xi$ and $\eta$ be two positive independent uncertain variables with regular uncertainty distributions $\Phi$ and $\Psi$, respectively. Then we have

$$E\left[\frac{\xi}{1+\eta}\right] = \int_0^1 \frac{\Phi^{-1}(\alpha)}{1+\Psi^{-1}(1-\alpha)}d\alpha.$$  

(7)

With the above concepts and theorems, we can model the pricing decision problem in uncertain environment.

3. Problem Description

We restrict our research on a supply chain in which a manufacturer distributes its product directly as well as through an independent retailer per market area. The manufacturer produces product at unit cost $\tilde{c}$ and wholesales the product to the retailer (r-channel) or sells directly to the consumers through an online shop (e-channel) with unit sales cost $\tilde{s}_e$. The retailer in turn retails the product to consumers with unit sales cost $\tilde{s}_r$. The pricing decision problem (how the manufacturer should set the wholesale price $w_r$ and sales price $p_e$ and how the retailer should make her retail markup policy $m_r$) in different power structures will be discussed. Some notations are listed in Table 1.

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
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<tr>
<td>$\tilde{c}$</td>
<td>unit manufacturing cost of the product, which is an uncertain variable.</td>
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<tr>
<td>$\tilde{s}_i$</td>
<td>unit sales cost of the product $i$ of the manufacturer, which is an uncertain variable.</td>
</tr>
<tr>
<td>$\tilde{s}_r$</td>
<td>unit sales cost of the product $i$ of the retailer, which is an uncertain variable.</td>
</tr>
<tr>
<td>$p_e$</td>
<td>unit direct sales price of the product, the manufacturer’s decision variable.</td>
</tr>
<tr>
<td>$w_r$</td>
<td>unit wholesale price of the product, the manufacturer’s decision variable.</td>
</tr>
<tr>
<td>$m_r$</td>
<td>unit retail margin of the product, the retailer’s decision variable</td>
</tr>
<tr>
<td>$p_r$</td>
<td>unit retail price of the product: $p_r = w_r + m_r$.</td>
</tr>
<tr>
<td>$\tilde{d}_e$</td>
<td>the market base of the online channel (e-channel), which is an uncertain variable.</td>
</tr>
<tr>
<td>$\tilde{d}_r$</td>
<td>the market base of the retail channel (r-channel), which is an uncertain variable.</td>
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In order to obtain the analytical solutions, as many scholars [McGuire and Staelin, 1983] [Choi, 1991] [Lee and Staelin, 1997] [Zhao et al., 2012], some reasonable assumptions are made.

Assumption 1. (Linear demand function) The target customers of the different channels may be different. When the channel prices change, they may change their choice. The customers make purchase decision based on the channel price itself as well as the comparison with the other alternative channel price. Hence, a linear price-dependent and channel-dependent demand function is built as follows:

$$q_i = \tilde{d}_i - \tilde{\beta} p_i + \tilde{\gamma} p_j$$

where $\tilde{d}_i$ is the market base of the channel $i$ (the potential market size with all the prices equaling 0), $\tilde{\beta}$ denotes the price elastic coefficient of channel $i$, $\tilde{\gamma}$ is the price elastic coefficient of channel $j$ (denoting substitutability of the two channels), and $p_i$ and $p_j$ are the sales prices of the channels $i$ and $j$, respectively. Note that changes in $\tilde{d}_i$ alter the relative channel preferences.
Because the channel demand should be more sensitive to changes in its price than to changes in the other channel price, it is assumed that the elastic coefficients $\beta$ and $\gamma$ satisfy

$$E[\hat{\beta}] > E[\hat{\gamma}] > 0.$$ (8)

**Assumption 2.** All the uncertain coefficients are assumed nonnegative and mutually independent.

**Assumption 3.** (Full information) The manufacturer and the retailer have the same information on the demands and the costs of the other channel member.

**Assumption 4.** (Risk neutral assumption) It is assumed that the manufacturer and the retailer are risk neutral and willing to maximize the expected profit.

Then the expected profit functions of the manufacturer and the retailer are as follows:

$$\pi_m = E[(p_e - \bar{c} - \bar{s}_e)(\bar{d}_e - \hat{\beta}p_e + \hat{\gamma}(m_e + w_e)) + (w_r - \bar{c})(\bar{d}_r - \beta(m_r + w_r) + \gamma p_r)],$$

$$\pi_r = E[(m_r - \bar{s}_r)(\bar{d}_r - \beta(m_r + w_r) + \gamma p_r)].$$ (9)

As the costs cannot exceed the retail price and markup, and the demands are always positive, then we have

**Assumption 5.** (Positive assumption) We assume that the costs cannot exceed the retail price and markup, and the demands are always positive.

$$M[w_r - \bar{c} \leq 0] = 0, \quad M[p_e - \bar{c} - \bar{s}_e \leq 0] = 0,$$

$$M[\bar{d}_r - \beta(m_r + w_r) + \gamma p_r \leq 0] = 0,$$

$$M[\bar{d}_e - \beta p_e + \gamma(m_r + w_r) \leq 0] = 0.$$ (10)

Let $\bar{c}, \bar{s}_e, \bar{s}_r, \bar{d}_e, \bar{d}_r, \hat{\beta}$ and $\hat{\gamma}$ be positive independent uncertain variables with regular uncertainty distributions $\Phi_{\bar{c}}, \Phi_{\bar{s}_e}, \Phi_{\bar{s}_r}, \Phi_{\bar{d}_e}, \Phi_{\bar{d}_r}, \Phi_{\hat{\beta}}$ and $\Phi_{\hat{\gamma}}$, respectively.

For concise, we define:

$$E[\tilde{a}^{1-\alpha} \tilde{b}^{\alpha}] = \int_0^1 \Phi_a^{-1}(1-\alpha)\Phi_b^{-1}(1-\alpha)dx,$$

$$E[\tilde{a}^{1-\alpha} \tilde{b}^{\alpha}] = \int_0^1 \Phi_a^{-1}(1-\alpha)\Phi_b^{-1}(1-\alpha)dx,$$

$$E[\tilde{a}^{1-\alpha} \tilde{b}^{\alpha}] = \int_0^1 \Phi_a^{-1}(1-\alpha)\Phi_b^{-1}(1-\alpha)dx,$$ (11)

where $\Phi_a^{-1}$ and $\Phi_b^{-1}$ is the reverse uncertainty distributions of uncertain variable $\tilde{a}$ and $\tilde{b}$, respectively.

**Proposition 1.** With the above assumptions, the expected profit functions of the manufacturer and the retailer are as follows:

$$\pi_m = -E[\tilde{\beta}] w_r^2 + (E[\tilde{d}_r] - E[\hat{\beta}] p_e + E[\tilde{\gamma}(\tilde{c}^{1-\alpha} + \tilde{s}_r^{1-\alpha})])w_r$$

$$+ 2E[\tilde{\gamma}] p_e w_r - E[\tilde{\beta}] p_e^2 + (E[\tilde{d}_e] + E[\hat{\gamma}] p_e) m_e + E[\tilde{\beta}^{1-\alpha}(\tilde{c}^{1-\alpha} + \tilde{s}_e^{1-\alpha})]$$

$$- E[\tilde{\gamma}^{1-\alpha}] p_e - E[\tilde{\gamma}] p_e^{1-\alpha}] + E[\tilde{d}_e^{1-\alpha} + \tilde{\gamma}] p_e m_e - E[\tilde{d}_e^{1-\alpha} + \tilde{s}_e^{1-\alpha})]$$

$$+ E[\tilde{\beta}^{1-\alpha}(\tilde{c}^{1-\alpha} + \tilde{s}_e^{1-\alpha})] m_e,$$

$$\pi_r = -[E[\hat{\beta}] m_r^2 + (E[\tilde{d}_r] - E[\hat{\gamma}] p_e) m_r + E[\tilde{\gamma}^{1-\alpha} \tilde{d}_r^{1-\alpha})] m_r$$

$$+ E[\tilde{d}_e^{\alpha} m_r^{1-\alpha})] - E[\tilde{\gamma}^{1-\alpha} \tilde{p}_e^{\alpha}) m_r.$$ (12)
Proof 1. If the conditions in Eq. (10) are satisfied, it is easy to see that \( \pi_m \) is monotone increasing with \( \tilde{d}, \tilde{d}_e, \tilde{y} \) and monotone decreasing with \( \bar{c}, \bar{s}, \bar{p}, \beta \), respectively. Then referring to Lemmas 2 and 3, it is obtained that

\[
E[(p_e - \bar{c} - \bar{s}_e)(\tilde{d}_e - \tilde{\beta}p_e + \bar{y}(m_r + w_r)) + (w_r - \bar{c})(\tilde{d}_e - \tilde{\beta}(m_r + w_r) + \bar{y}p_e)]
= \int_0^1 [(w_r - \Phi_c^{-1}(1 - \alpha))(\Phi_c^{-1}(\alpha) - \Phi_{\tilde{\beta}}^{-1}(1 - \alpha)m_r + w_r) + \Phi_{\bar{y}}^{-1}(\alpha)p_e]d\alpha

+ (p_e - \Phi_{\tilde{\beta}}^{-1}(1 - \alpha) - \Phi_{\tilde{\beta}}^{-1}(1 - \alpha))(\Phi_{\tilde{\beta}}^{-1}(\alpha) - \Phi_{\bar{y}}^{-1}(1 - \alpha)p_e + \Phi_{\bar{y}}^{-1}(\alpha)(m_r + w_r))d\alpha
= w_rE[\tilde{d}_e] - E[\tilde{\beta}m_r + w_r]w_r + E[\tilde{y}]p_ep_e - E[\tilde{d}_e\tilde{\beta}^{-1\alpha}](w_r + m_r)
- E[\tilde{d}_e\tilde{\beta}^{-1\alpha}]p_e + E[\tilde{d}_e]\tilde{\beta}^{-1\alpha}p_e + E[\tilde{y}](m_r + w_r)p_e - E[\tilde{d}_e\tilde{\beta}^{-1\alpha}](\tilde{\beta}^{-1\alpha} + \tilde{s}_e^{-1\alpha})]
+ E[\tilde{\beta}^{-1\alpha}(\tilde{\beta}^{-1\alpha} + \tilde{s}_e^{-1\alpha})]p_e - E[\tilde{\beta}^{-1\alpha}(\tilde{\beta}^{-1\alpha} + \tilde{s}_e^{-1\alpha})](w_r + m_r)

= -E[\tilde{\beta}]w_r^2 + (E[\tilde{d}_e] - E[\tilde{\beta}]m_r + E[\tilde{\beta}^{-1\alpha}m_r] - E[\tilde{\beta}^{-1\alpha}\tilde{\beta}^{-1\alpha}])w_r + 2E[\tilde{\beta}]p_ep_e
- E[\tilde{\beta}]p_e^2 + (E[\tilde{d}_e] + E[\tilde{y}]m_r + E[\tilde{\beta}^{-1\alpha}(\tilde{\beta}^{-1\alpha} + \tilde{s}_e^{-1\alpha})])p_e
- E[\tilde{d}_e\tilde{\beta}^{-1\alpha}]m_r - E[\tilde{d}_e\tilde{\beta}^{-1\alpha}m_r] - E[\tilde{d}_e\tilde{\beta}^{-1\alpha}(\tilde{\beta}^{-1\alpha} + \tilde{s}_e^{-1\alpha})]m_r.

In the same way, when conditions \( \mathcal{M}[\tilde{d}_e, -\beta(m_r + w_r) + \gamma p_e \leq 0] = 0 \) and \( \mathcal{M}[m_r - \bar{s}_e \leq 0] = 0 \) hold, \( \pi_r \) is as follows:

\[
\pi_r = \int_0^1 (m_r - \Phi_{\bar{y}}^{-1}(1 - \alpha))(\Phi_{\bar{y}}^{-1}(\alpha) - \Phi_{\bar{\beta}}^{-1}(1 - \alpha)m_r + w_r) + \Phi_{\bar{s}}^{-1}(\alpha)p_e)d\alpha
= m_rE[\tilde{d}_e] - E[\tilde{\beta}]m_r^2 + E[\tilde{\beta}]w_r m_r + E[\tilde{y}]p_em_r - E[\tilde{d}_e\tilde{\beta}^{-1\alpha}]
+ E[\tilde{d}_e\tilde{\beta}^{-1\alpha}m_r + w_r] - E[\tilde{d}_e\tilde{\beta}^{-1\alpha}]p_e

= -E[\tilde{\beta}]m_r^2 + (E[\tilde{d}_e] - E[\tilde{\beta}]w_r + E[\tilde{y}]p_em_r + E[\tilde{d}_e\tilde{\beta}^{-1\alpha}]m_r)
+ E[\tilde{d}_e\tilde{\beta}^{-1\alpha}] - E[\tilde{d}_e\tilde{\beta}^{-1\alpha}]p_e.
\]

Proposition 6 is proved.

4. Equilibrium Analysis

In this section, we will discuss the equilibrium behaviors of the supply chain participants in markets with different power structures. Moreover, we assume that the power structures are represented by the different sequences in which the wholesale prices and retail margins are chosen by the participants, and the powerful participants move first and perform as Stackelberg leader. The key difference of the three possible power structures is the move sequence of the participants.

4.1. Manufacturer-Stackelberg Case

In some supply chains, the manufacturer is much larger than the retailer in each market and he can dominate the supply chain with a first-move advantage over the retailer. Therefore, he performs as Stackelberg leader and the
In order to solve the Stackelberg model, we should derive the follower’s optimal response to the given (\( \pi \) equaling zero. According to the assumption that \( w_r - \tilde{c} \leq 0 \), we can get the crisp form of \( \pi_r \), differentiate the equation and then set the first-order derivative equaling zero.

\[
\frac{\partial E[\pi_r]}{\partial m_r} = -2E[\tilde{\beta}]m_r + E[\tilde{d}_r] - E[\tilde{\beta}]w_r + E[\tilde{\gamma}]p_e + E[\tilde{\gamma}_1\tilde{\alpha}\tilde{\beta}^{1-\alpha}] = 0.
\]

We can obtain the retailer’s response to the manufacturer’s decision \((w_r, p_e)\) by solving the above equation.

\[
m_r^* = \frac{E[\tilde{d}_r] - E[\tilde{\beta}]w_r + E[\tilde{\gamma}]p_e + E[\tilde{\gamma}_1\tilde{\alpha}\tilde{\beta}^{1-\alpha}]}{2E[\tilde{\beta}]}. \tag{17}
\]

Then substituting \( m_r^* \) into the manufacturer’s profit function and differentiating it, we can get second-order derivatives as follows:

\[
\frac{\partial^2 \pi_m(w_r, p_e)}{\partial w^2_r} = -E[\tilde{\beta}]; \quad \frac{\partial^2 \pi_m(w_r, p_e)}{\partial w_r \partial p_e} = E[\tilde{\gamma}]; \quad \frac{\partial^2 \pi_m(w_r, p_e)}{\partial p^2_e} = -2E[\tilde{\beta}]^2 + E[\tilde{\gamma}_1]^2 \tag{18}
\]

Then the Hessian matrix can be attained

\[
H_1 = \begin{bmatrix}
\frac{\partial^2 \pi_m(w_r, p_e)}{\partial w^2_r} & \frac{\partial^2 \pi_m(w_r, p_e)}{\partial w_r \partial p_e} \\
\frac{\partial^2 \pi_m(w_r, p_e)}{\partial p^2_e}
\end{bmatrix} = \begin{bmatrix}
-E[\tilde{\beta}] & E[\tilde{\gamma}] \\
E[\tilde{\gamma}] & -2E[\tilde{\beta}]^2 + E[\tilde{\gamma}_1]^2
\end{bmatrix} \tag{19}
\]

According to the assumption that \( E[\tilde{\beta}] > E[\tilde{\gamma}] > 0 \), then \( -\frac{2E[\tilde{\beta}]^2 + E[\tilde{\gamma}_1]^2}{E[\tilde{\beta}]} < -\frac{2E[\tilde{\beta}]^2 + E[\tilde{\gamma}_1]^2}{E[\tilde{\beta}]} = -E[\tilde{\beta}] < -E[\tilde{\gamma}] \). Thus, \( H_1 \) is a definite negative matrix and \( \pi_m(p_e, w_r) \) is jointly concave in \( p_e \) and \( w_r \). Therefore, we can obtain the optimal solutions by setting the first-order derivatives as

\[
\frac{\partial \pi_m}{\partial w_r} = -E[\tilde{\beta}]w_r + E[d_r] - E[\tilde{\beta}^{1-\alpha}\tilde{\gamma}_1^{1-\alpha}] + E[\tilde{\gamma}^{1-\alpha}\tilde{\beta}^{1-\alpha}] - \frac{E[(\tilde{\gamma}_1^{1-\alpha} + \tilde{\beta}^{1-\alpha})\tilde{\gamma}]}{2} + E[\tilde{\gamma}]p_e = 0;
\]

\[
\frac{\partial \pi_m}{\partial p_e} = -2E[\tilde{\beta}]^2 + E[\tilde{\gamma}_1]^2 p_e + E[d_r] + E[\tilde{\gamma}]E[\tilde{d}_r] + E[\tilde{\gamma}_1^{1-\alpha}\tilde{\beta}^{1-\alpha}]E[\tilde{\gamma}] + E[\tilde{\gamma}_1^{1-\alpha}\tilde{\beta}^{1-\alpha}]E[\tilde{\gamma}] + E[\tilde{\gamma}_1^{1-\alpha}\tilde{\beta}^{1-\alpha}]E[\tilde{\gamma}]
\]

\[
+ \frac{E[\tilde{\gamma}_1^{1-\alpha}\tilde{\beta}^{1-\alpha}]E[\tilde{\gamma}]}{2E[\tilde{\beta}]} - E[\tilde{\gamma}_1^{1-\alpha}\tilde{\gamma}^\prime] + E[\tilde{\gamma}]w_r = 0. \tag{20}
\]
The optimal decision of the manufacturer can be attained by solving the above two equations as follows:

\[ p^*_c = \frac{E[\bar{\beta}] w^*_c + E[\bar{\beta}] E[\bar{\beta}^{1-\alpha} (c^{1-\alpha} + \tilde{s}_c^{1-\alpha})] - E[\bar{\gamma}] E[\bar{\gamma}^\alpha (c^{1-\alpha} + \tilde{s}_c^{1-\alpha})]}{2(E[\bar{\beta}]^2 - E[\bar{\gamma}]^2)} + \frac{E[\bar{\gamma}] E[\bar{\beta}^{1-\alpha} (c^{1-\alpha} + \tilde{s}_c^{1-\alpha})] - E[\bar{\beta}] E[\bar{\beta}^{1-\alpha} (c^{1-\alpha} + \tilde{s}_c^{1-\alpha})]}{2(E[\bar{\beta}]^2 - E[\bar{\gamma}]^2)}; \]

\[ w^*_r = \frac{E[d_r] - E[\bar{\beta}^{1-\alpha} \tilde{s}_c^{1-\alpha}]}{2E[\bar{\beta}]} \frac{E[\beta] E[\bar{\beta}^{1-\alpha} (c^{1-\alpha} + \tilde{s}_c^{1-\alpha})] + E[\bar{\gamma}] p^*_c}{E[\bar{\beta}]^2} \]

Evidently, the retailer’s decision can be easily attained

\[ m^*_r = \frac{E[d_r] - E[\bar{\beta}] w^*_r + E[\bar{\gamma}] p^*_c + E[\bar{\beta}^{1-\alpha} (c^{1-\alpha} + \tilde{s}_c^{1-\alpha})]}{2E[\bar{\beta}]}. \]

4.2. Retailer-Stackelberg Case

As many studies in marketing and distribution channels show, the dominance relationship between manufacturers and retailers has reversed. In many industries, manufacturers might have to face a super retailer like Warmart or Carrefour, who holds most of the power in the supply chain.

\[
\begin{align*}
\text{max } & \pi_r = E[(m_r - \tilde{s}_c)(d_r - \bar{\beta}(m_r + w^*_r)\tilde{\gamma} p^*_c)] \\
\text{subject to:} & \mathcal{N}(m_r - \tilde{s}_c \leq 0) = 0 \\
\end{align*}
\]

where \((w^*_r, p^*_c)\) derives from problem:

\[
\begin{align*}
\text{max } & \pi_m = E[(p_c - \tilde{c} - \tilde{s}_c)(d_r - \bar{\beta} p_c + \tilde{\gamma}(m_r + w_r)) + (w_r - \tilde{c})(d_r - \bar{\beta}(m_r + w_r) + \gamma p_c)] \\
\text{subject to:} & \mathcal{N}(w_r - \tilde{c} \leq 0) = 0, \mathcal{N}(p_c - \tilde{c} - \tilde{s}_c \leq 0) = 0, \mathcal{N}(d_r - \bar{\beta} p_c + \tilde{\gamma}(m_r + w_r) \leq 0) = 0, \mathcal{N}(d_r - \bar{\beta} p_c + \tilde{\gamma}(m_r + w_r) \leq 0) = 0.
\end{align*}
\]

Referring to Eq. (13), we can achieve the crisp profit function of the manufacturer \(\pi_m(w_r, p_c)\) and the Hessian matrix

\[ H_2 = \begin{bmatrix}
\frac{\partial^2 \pi_m(w_r, p_c)}{\partial^2 w_r} & \frac{\partial^2 \pi_m(w_r, p_c)}{\partial w_r \partial p_c} \\
\frac{\partial^2 \pi_m(w_r, p_c)}{\partial w_r \partial p_c} & \frac{\partial^2 \pi_m(w_r, p_c)}{\partial^2 p_c}
\end{bmatrix} = \begin{bmatrix}
-2E[\bar{\beta}] & 2E[\bar{\gamma}] \\
-2E[\bar{\gamma}] & -2E[\bar{\beta}]
\end{bmatrix} \]

Note that \(E[\bar{\beta}] > E[\bar{\gamma}] > 0\). It can easily be seen that the hessian matrix \(H_2\) is negative definite with respect to \((w_r, p_c)\). Thus \(\pi_m(w_r, p_c)\) is concave with \(p_c\) and \(w_r\). Holding the constraint conditions of the model and letting the first-order derivatives equaling zero, it can be obtained as follows:

\[
\begin{align*}
\frac{\partial \pi_m(w_r, p_c)}{\partial w_r} &= -2E[\bar{\beta}] w_r + E[d_r] - E[\bar{\beta}] m_r + E[\bar{\beta}^{1-\alpha} (c^{1-\alpha} + \tilde{s}_c^{1-\alpha})] \\
\text{and } E[\bar{\gamma}^{\alpha} (\tilde{c} + \tilde{s}_c)^{1-\alpha}] + 2E[\bar{\gamma}] p_c = 0; \\
\frac{\partial \pi_m(w_r, p_c)}{\partial p_c} &= -2E[\bar{\beta}] p_c + E[d_r] + E[\bar{\gamma}] m_r + E[\bar{\beta}^{1-\alpha} (\tilde{c} + \tilde{s}_c)^{1-\alpha}] \\
\text{and } E[\bar{\gamma}^{\alpha} (\tilde{c}^{1-\alpha}) + 2E[\bar{\gamma}] w_r = 0.
\end{align*}
\]
By solving Eq. (25) with respect to \((w_r, p_r)\), the manufacturer’s optimal response can be obtained as follows:

\[
p_r^* = \frac{E[\tilde{\beta}]E[\tilde{d}_r] + E[\tilde{\beta}]E[\tilde{\beta}^{1-a}(\hat{c} + s_r)^{1-a}] - E[\tilde{\beta}]E[\tilde{\gamma}\hat{c}^{1-a}]}{2(E[\tilde{\beta}]^2 - E[\tilde{\gamma}]^2)} + \frac{E[\tilde{\gamma}]E[\tilde{d}_r] + E[\tilde{\gamma}]E[\tilde{\beta}^{1-a}(\hat{c} + s_r)^{1-a}] - E[\tilde{\gamma}]E[\tilde{\gamma}\hat{c}^{1-a}]}{2(E[\tilde{\beta}]^2 - E[\tilde{\gamma}]^2)},
\]

\[
w_r^* = \frac{E[\tilde{\gamma}]E[\tilde{d}_r] + E[\tilde{\gamma}]E[\tilde{\beta}^{1-a}(\hat{c} + s_r)^{1-a}] - E[\tilde{\gamma}]E[\tilde{\gamma}\hat{c}^{1-a}]}{2(E[\tilde{\beta}]^2 - E[\tilde{\gamma}]^2) m_r}.
\]

Substituting the response into the profit function of the retailer, we have

\[
\pi_r = -\frac{1}{2}E[\tilde{\beta}]m_r^2 + E[\tilde{d}_r]m_r - E[\tilde{\beta}]Wm_r + E[\tilde{\gamma}]Pm_r + \frac{1}{2}E[\tilde{\beta}^{1-a}\tilde{s}_r^{1-a}]m_r - E[\hat{c}^{1-a}\tilde{s}_r^{1-a}]W + E[\hat{c}\tilde{s}_r^{1-a}]P
\]

where

\[
P = \frac{E[\tilde{\beta}]E[\tilde{d}_r] + E[\tilde{\beta}]E[\tilde{\beta}^{1-a}(\hat{c} + s_r)^{1-a}] - E[\tilde{\beta}]E[\tilde{\gamma}\hat{c}^{1-a}]}{2(E[\tilde{\beta}]^2 - E[\tilde{\gamma}]^2)} + \frac{E[\tilde{\gamma}]E[\tilde{d}_r] + E[\tilde{\gamma}]E[\tilde{\beta}^{1-a}(\hat{c} + s_r)^{1-a}] - E[\tilde{\gamma}]E[\tilde{\gamma}\hat{c}^{1-a}]}{2(E[\tilde{\beta}]^2 - E[\tilde{\gamma}]^2)}
\]

\[
W = \frac{E[\tilde{\gamma}]E[\tilde{d}_r] + E[\tilde{\gamma}]E[\tilde{\beta}^{1-a}(\hat{c} + s_r)^{1-a}] - E[\tilde{\gamma}]E[\tilde{\gamma}\hat{c}^{1-a}]}{2(E[\tilde{\beta}]^2 - E[\tilde{\gamma}]^2)}
\]

Because the second-order derivative

\[
\frac{\partial^2 E[\pi_r(m_r)]}{\partial m_r^2} = -E[\tilde{\beta}] < 0,
\]

setting

\[
\frac{\partial E[\pi_r(m_r)]}{\partial m_r} = -E[\tilde{\beta}]m_r + E[\tilde{d}_r] - E[\tilde{\beta}]W + E[\tilde{\gamma}]P + \frac{1}{2}E[\tilde{\beta}^{1-a}\tilde{s}_r^{1-a}] = 0
\]

and solving Eq. (30), we can attain the optimal retail markup price of the retailer

\[
m_r^* = \frac{E[\tilde{d}_r] - E[\tilde{\beta}]W + E[\tilde{\gamma}]P + \frac{1}{2}E[\tilde{\beta}^{1-a}\tilde{s}_r^{1-a}]}{E[\tilde{\beta}]}.
\]

4.3. Vertical-Nash Case

The last case occurs in markets where the manufacturer and the retailer have equal bargaining power and make decisions simultaneously. In this decentralized structure, each participant, faced with a downward sloping demand curve and given predicted costs, non-cooperatively makes its pricing decision to maximize its expected profit given the competitor’s decision. Hence, a Nash game model can be applied as

\[
\begin{align*}
\max_{m, \bar{m}} \pi_r & = E[(m_r - \hat{s}_r)(\tilde{d}_r - \tilde{\beta}(m_r + w_r) + \tilde{\gamma} p_r)] \\
\max_{w_r, p_r} \pi_m & = E[(p_r - \hat{\ell}_r)(\tilde{d}_r - \tilde{\beta} p_r + \tilde{\gamma}(m_r + w_r)) + (w_r - \hat{\ell}_r)(\tilde{d}_r - \tilde{\beta}(m_r + w_r) + \tilde{\gamma} p_r)]
\end{align*}
\]

subject to:

\[
\begin{align*}
\mathcal{M}(m_r - \hat{s}_r \leq 0) & = 0, \quad \mathcal{M}(w_r - \hat{\ell}_r \leq 0) = 0, \quad \mathcal{M}(p_r - \hat{\ell}_r \leq 0) = 0, \\
\mathcal{M}(\tilde{d}_r - \tilde{\beta}(m_r + w_r) + \tilde{\gamma} p_r \leq 0) & = 0, \quad \mathcal{M}(\tilde{d}_r - \tilde{\beta} p_r + \tilde{\gamma}(m_r + w_r) \leq 0) = 0.
\end{align*}
\]
To solve this model, given the manufacturer’s decision \((w_r, p_r)\), the optimal response of the retailer is as follows:

\[
m_r = \frac{E[\hat{d}_r] - E[\hat{\beta}]w_r + E[\hat{\gamma}]p_r + E[\hat{s}_r] E[\hat{\beta}_r]}{2E[\hat{\beta}]}. \tag{33}
\]

Once again, given the retailer’s markup policy \(m_r\), then the optimal response of the manufacturer can be attained easily.

\[
p_r = \frac{E[\hat{\beta}]E[\hat{d}_r] + E[\hat{\beta}]E[\hat{\beta}_r (\hat{\gamma} + \hat{s}_r)] - E[\hat{\beta}]E[\hat{\beta}_r (\hat{\gamma} + \hat{s}_r)]}{2E[\hat{\beta}^2 - E[\hat{\gamma}^2]} \tag{34}
\]

\[
w_r = \frac{E[\hat{\beta}]E[\hat{d}_r] + E[\hat{\beta}]E[\hat{\beta}_r (\hat{\gamma} + \hat{s}_r)] - E[\hat{\beta}]E[\hat{\beta}_r (\hat{\gamma} + \hat{s}_r)]}{2E[\hat{\beta}^2 - E[\hat{\gamma}^2]} \tag{35}
\]

Combining Eqs. (33) and (34) and solving these equations, we have

\[
m_r^* = \frac{E[\hat{d}_r] + 2E[\hat{\beta}_r] E[\hat{\beta}_r] - E[\hat{\beta}_r] E[\hat{\beta}_r] + E[(\hat{\beta}_r^2 + \hat{s}_r)] E[\hat{\beta}_r]}{3E[\hat{\beta}]}.
\]

\[
p_r^* = \frac{E[\hat{\beta}]E[\hat{d}_r] + E[\hat{\beta}]E[\hat{\beta}_r (\hat{\gamma} + \hat{s}_r)] - E[\hat{\beta}]E[\hat{\beta}_r (\hat{\gamma} + \hat{s}_r)]}{2E[\hat{\beta}^2 - E[\hat{\gamma}^2} \tag{35}
\]

\[
w_r^* = \frac{E[\hat{d}_r] - E[\hat{\beta}_r (\hat{\gamma} + \hat{s}_r)] + 2E[\hat{\beta}_r (\hat{\gamma} + \hat{s}_r)] - 2E[(\hat{\beta}_r^2 + \hat{s}_r)] E[\hat{\beta}_r]}{3E[\hat{\beta}]}.
\]

5. Numerical Experiments

Owing to the complicated forms of the equilibrium prices and expected profits, it is very difficult (if possible) to make analytical comparisons. Therefore, without loss of generality, we conduct a series of numerical experiments to explore the effects of the different power structures and parameters’ uncertain degrees on optimal equilibrium prices. As the results from different experiments are somewhat coherent, we present only one of them in this part.

We consider a case where a manufacturer produces a new product and then distributes it through an e-channel and a traditional r-channel as well. As the product has never been sold as well as the changeable environments, the participants have no historical data when making their pricing decisions. Hence applying specialists’ or market managers’ predictions can truly make a difference. Interested readers can consult Liu (2010) (Chapter 4) to get more detailed about how to obtain expert’s experimental data and how to determine the uncertainty distribution for an uncertain variable through expert’s experimental data. For simplicity, we just present the corresponding uncertain distributions for the manufacturing cost, sales costs and market bases in Table 2.

Referring to Lemma 2, the expected values of the uncertain variables can be derived from their distribution functions.

\[
E[\hat{\beta}_r] = \frac{3000 + \frac{2}{4} \cdot 3600 + \frac{2}{4} \cdot 4000}{4} = 3550, \quad E[\hat{\beta}_r] = \frac{4000 + \frac{2}{4} \cdot 5000 + \frac{2}{4} \cdot 5500}{4} = 4875.
\]
Table 2: Uncertain variables

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Distribution</th>
<th>Expected value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{c} )</td>
<td>( \mathcal{U}(9, 11) )</td>
<td>10</td>
</tr>
<tr>
<td>( \hat{s}_e )</td>
<td>( \mathcal{U}(3, 5) )</td>
<td>4</td>
</tr>
<tr>
<td>( \hat{s}_r )</td>
<td>( \mathcal{U}(5, 7) )</td>
<td>6</td>
</tr>
<tr>
<td>( \hat{\beta} )</td>
<td>( \mathcal{U}(180, 220) )</td>
<td>200</td>
</tr>
<tr>
<td>( \hat{\gamma} )</td>
<td>( \mathcal{U}(90, 110) )</td>
<td>100</td>
</tr>
<tr>
<td>( \hat{d}_e )</td>
<td>( \mathcal{Z}(3000, 3600, 4000) )</td>
<td>3550</td>
</tr>
<tr>
<td>( \hat{d}_r )</td>
<td>( \mathcal{Z}(4000, 5000, 5500) )</td>
<td>4875</td>
</tr>
</tbody>
</table>

According to Lemma 3, we have

\[
E[\hat{c}^{1-\alpha}\hat{d}_e^{\alpha}] = \int_0^1 \Phi^{-1}_c(1-\alpha)\Phi^{-1}_d(\alpha)\,d\alpha
\]

\[
= \int_0^{0.5} \left[ (5000 \times (2-2(1-\alpha)) + 5500 \times 2(1-\alpha))(9 + (11 - 9)\alpha) \right. \,d\alpha
\]

\[
+ \int_0^{1} \left( 4000 \times (1 - 2(1 - \alpha)) + 5000 \times (2(1-\alpha) - 1))(9 + (11 - 9)\alpha) \right. \,d\alpha
\]

\[
= 36000
\]  

(36)

In the same way, we can obtain the value of \( E[\hat{c}^{1-\alpha}\hat{d}_r^{\alpha}] \), \( E[\hat{c}^{1-\alpha}\hat{\beta}^{1-\alpha}] \), \( E[\hat{c}^{1-\alpha}\hat{\gamma}^{\alpha}] \), \( E[\hat{s}_e^{1-\alpha}\hat{\beta}^{1-\alpha}] \), \( E[\hat{s}_r^{1-\alpha}\hat{\gamma}^{\alpha}] \), etc.

5.1. Effects of the Power Structures

By using the model above, the equilibrium prices and expected profits of the participants under the three different power structures can be attained in Table 3.

Table 3: The equilibrium prices and expected profits

<table>
<thead>
<tr>
<th>Structures</th>
<th>( w_r^* )</th>
<th>( p_r^* )</th>
<th>( E[\pi_m] )</th>
<th>( m_r^* )</th>
<th>( p_r^* )</th>
<th>( E[\pi_r] )</th>
<th>( E[\pi_t] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS</td>
<td>24.2222</td>
<td>27.0361</td>
<td>27342.50</td>
<td>9.8521</td>
<td>34.0743</td>
<td>11375.98</td>
<td>38718.48</td>
</tr>
<tr>
<td>VN</td>
<td>21.6764</td>
<td>27.0361</td>
<td>26694.38</td>
<td>11.1250</td>
<td>32.8014</td>
<td>13644.42</td>
<td>40338.80</td>
</tr>
<tr>
<td>RS</td>
<td>20.4035</td>
<td>27.0361</td>
<td>24425.93</td>
<td>13.6708</td>
<td>34.0743</td>
<td>14292.55</td>
<td>38718.48</td>
</tr>
</tbody>
</table>

As we can see from Table 3 (or other experiments in the following part), the sales price of e-channel in the three structures are the same. The expected profits of the total system \( E[\pi_t] \) in the two Stackelberg cases are equal, indicating that who holds the power has no influence on the total supply chain. But from the prospect of the individual firms, the firm as a leader will gain more profit than as a follower. Additionally, the supply chain can gain more profit and the consumers can enjoy lower sales prices \( m_r \) when there is no dominant participant in the system.

5.2. Effects of the Uncertain Degrees

In this part, we will analyze the effects of the uncertain degrees of the costs including \( \hat{c} \), \( \hat{s}_e \) and \( \hat{s}_r \) on the channel member’s pricing decisions. The uncertain degrees of the parameters mainly depend on experts’ personal knowledge about the parameters. More information about the parameters is available, more accurate estimations the experts can make, and as a result the uncertain degrees of the parameters will become lower.

By changing the uncertain degrees of corresponding costs and keeping other parameters the same as shown in Table 2. The results of optimal prices and profits are shown in Table 4,5,6.

From Table 4, we can find that:

- Regardless of the power structure, the wholesale price and sales price in the r-channel as well as the sales prices in the e-channel will rise up when the uncertain degree of the manufacturing cost increases. However, the markup price charged by the retailer in r-channel will drop slightly.
Table 4: The equilibrium prices and expected profits with various uncertain degrees of $\tilde{c}$

<table>
<thead>
<tr>
<th>Structure</th>
<th>$\tilde{c}$</th>
<th>$w^*_c$</th>
<th>$p^*_r$</th>
<th>$E[\pi_m]$</th>
<th>$m^*_r$</th>
<th>$p^*_r$</th>
<th>$E[\pi_r]$</th>
<th>$E[\pi_t]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS</td>
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<td>24.1722</td>
<td>26.9861</td>
<td>26315.17</td>
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<tr>
<td></td>
<td>L(9,11)</td>
<td>24.2222</td>
<td>27.0361</td>
<td>27342.50</td>
<td>9.8521</td>
<td>34.0743</td>
<td>11375.98</td>
<td>38718.48</td>
</tr>
<tr>
<td></td>
<td>L(8,12)</td>
<td>24.2722</td>
<td>27.1361</td>
<td>28370.71</td>
<td>9.8396</td>
<td>34.1493</td>
<td>11357.25</td>
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</tr>
<tr>
<td></td>
<td>L(7,13)</td>
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<tr>
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<td>L(9,11)</td>
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<td>RS</td>
<td>L(10,10)</td>
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<td>14217.09</td>
<td>40738.38</td>
</tr>
</tbody>
</table>

- In the three power structures, the expected profits of the manufacturer will increase when the uncertain degree of $\tilde{c}$ rise up, whereas the retailer will suffer from lower profit.

Table 5: The equilibrium prices and expected profits with various uncertain degrees of $\tilde{s}$

<table>
<thead>
<tr>
<th>Structure</th>
<th>$\tilde{c}$</th>
<th>$w^*_c$</th>
<th>$p^*_r$</th>
<th>$E[\pi_m]$</th>
<th>$m^*_r$</th>
<th>$p^*_r$</th>
<th>$E[\pi_r]$</th>
<th>$E[\pi_t]$</th>
</tr>
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<tbody>
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<td>27342.50</td>
<td>9.8521</td>
<td>34.0743</td>
<td>11375.98</td>
<td>38718.48</td>
</tr>
<tr>
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<td>L(2,6)</td>
<td>24.2444</td>
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<td>27886.45</td>
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<td>34.0743</td>
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</tr>
<tr>
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<td>L(3,5)</td>
<td>21.6764</td>
<td>27.0361</td>
<td>26694.38</td>
<td>13.6458</td>
<td>34.1118</td>
<td>14254.76</td>
<td>39727.96</td>
</tr>
<tr>
<td></td>
<td>L(2,6)</td>
<td>21.7014</td>
<td>27.0639</td>
<td>27239.73</td>
<td>13.6208</td>
<td>34.1493</td>
<td>14217.09</td>
<td>40738.38</td>
</tr>
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<td>L(1,7)</td>
<td>21.7264</td>
<td>27.0917</td>
<td>27785.34</td>
<td>13.6199</td>
<td>34.1493</td>
<td>14217.09</td>
<td>40738.38</td>
</tr>
<tr>
<td>RS</td>
<td>L(4,4)</td>
<td>20.3771</td>
<td>27.0083</td>
<td>23875.87</td>
<td>13.6708</td>
<td>34.0743</td>
<td>14305.10</td>
<td>38180.96</td>
</tr>
<tr>
<td></td>
<td>L(3,5)</td>
<td>20.4035</td>
<td>27.0361</td>
<td>24425.93</td>
<td>13.6708</td>
<td>34.0743</td>
<td>14292.55</td>
<td>38718.48</td>
</tr>
<tr>
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<td>L(2,6)</td>
<td>20.4299</td>
<td>27.0639</td>
<td>24976.24</td>
<td>13.6625</td>
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<td>L(1,7)</td>
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<td>27.0917</td>
<td>25526.78</td>
<td>13.6542</td>
<td>34.1104</td>
<td>14267.49</td>
<td>39794.27</td>
</tr>
</tbody>
</table>

- From Table 5 we can find that:
  - The wholesale price and sales price in the r-channel and the sales prices in the e-channel will increase respectively when the uncertain degree of the sales cost in e-channel rise up. Unlikely, the markup price of the r-channel, which determined by the retailer, will decline slightly when it increases.
  - In the similar way, the manufacturer can benefit from the vagueness of the sales costs of the e-channel, whereas the retailer in e-channel will suffer from lower profit when the uncertain degree of $\tilde{s}$ increase.

From Table 6, we can find that:
- Contrary to the above results, the wholesale prices of the manufacturer will decrease with the uncertain degree of $\tilde{s}$. The sales prices in the e-channel keep unchanged indicating that the uncertain degree of the sales costs in r-channel has no influence on the sales prices in other channel. Additionally, the retailer can charge higher markup which in turn lead to a higher sales prices in the r-channel when the sales costs’ uncertain degree rise up.
- Regardless of the power structure, the retailer will gain more and the manufacturer will gain less when the uncertain degree of $\tilde{s}$ increase.

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Table 6: The equilibrium prices and expected profits with various uncertain degrees of $\tilde{c}$, $\tilde{s}$, $\tilde{c}_r$ and $\tilde{s}_r$

<table>
<thead>
<tr>
<th>Structure</th>
<th>$\tilde{c}$</th>
<th>$w^*_c$</th>
<th>$p^*_c$</th>
<th>$E[\pi_m]$</th>
<th>$m^*_c$</th>
<th>$p^*_r$</th>
<th>$E[\pi_r]$</th>
<th>$E[\pi_t]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS</td>
<td>$\mathcal{L}(6, 6)$</td>
<td>24.2389</td>
<td>27.0361</td>
<td>27367.99</td>
<td>9.8271</td>
<td>34.0660</td>
<td>10879.31</td>
<td>38247.30</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{L}(5, 7)$</td>
<td>24.2222</td>
<td>27.0361</td>
<td>27342.50</td>
<td>9.8521</td>
<td>34.0743</td>
<td>11375.98</td>
<td>38718.48</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{L}(4, 8)$</td>
<td>24.2056</td>
<td>27.0361</td>
<td>27317.07</td>
<td>9.8771</td>
<td>34.0826</td>
<td>11872.67</td>
<td>39189.74</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{L}(3, 9)$</td>
<td>24.1889</td>
<td>27.0361</td>
<td>27291.70</td>
<td>9.9021</td>
<td>34.0910</td>
<td>12369.39</td>
<td>39661.09</td>
</tr>
<tr>
<td>VN</td>
<td>$\mathcal{L}(6, 6)$</td>
<td>21.6875</td>
<td>27.0361</td>
<td>26717.03</td>
<td>11.1028</td>
<td>32.7903</td>
<td>13157.67</td>
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<tr>
<td></td>
<td>$\mathcal{L}(5, 7)$</td>
<td>21.6764</td>
<td>27.0361</td>
<td>26694.38</td>
<td>11.1250</td>
<td>32.8014</td>
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<td>$\mathcal{L}(4, 8)$</td>
<td>21.6653</td>
<td>27.0361</td>
<td>26671.77</td>
<td>11.1472</td>
<td>32.8125</td>
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</tr>
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<td>$\mathcal{L}(3, 9)$</td>
<td>21.6542</td>
<td>27.0361</td>
<td>26649.22</td>
<td>11.1694</td>
<td>32.8236</td>
<td>14618.08</td>
<td>41267.29</td>
</tr>
<tr>
<td>RS</td>
<td>$\mathcal{L}(6, 6)$</td>
<td>20.4118</td>
<td>27.0361</td>
<td>24438.68</td>
<td>13.6542</td>
<td>34.0660</td>
<td>13808.63</td>
<td>38247.30</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{L}(5, 7)$</td>
<td>20.4035</td>
<td>27.0361</td>
<td>24425.93</td>
<td>13.6708</td>
<td>34.0743</td>
<td>14292.55</td>
<td>38718.48</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{L}(4, 8)$</td>
<td>20.3951</td>
<td>27.0361</td>
<td>24413.22</td>
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<td>$\mathcal{L}(3, 9)$</td>
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<td>13.7042</td>
<td>34.0910</td>
<td>15260.56</td>
<td>39661.09</td>
</tr>
</tbody>
</table>

From the above results, one can also conclude that when the uncertain degrees of the some channel members’ costs ($\tilde{c}$, $\tilde{s}$ for the manufacturer and $\tilde{c}_r$ for the retailer) increase, which indicates that little information about them is available for the decision makers (of course the other channel members), the ones can charge a higher prices and gain more profits.

We may concern that weather the change of parameters $\tilde{\beta}$ and $\tilde{\gamma}$ might affect the conclusions. Note that the influence of the uncertain degrees of the costs mainly decided by the ratio of the two price elasticity which represent the competing intensity or substitutability of the two channels. By keeping $\tilde{\beta}$ unchanged and varying $\tilde{\gamma}$ in three levels as shown in Table 7. However, the same with other experiments, the result keep the coherent as shown above.

Table 7: Different competing intensity

<table>
<thead>
<tr>
<th>Parameters Competing Intensity</th>
<th>Distribution</th>
<th>Expected value $E[\tilde{\gamma}]/E[\tilde{\beta}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\gamma}$ low</td>
<td>$\mathcal{L}(40, 60)$</td>
<td>50 0.25</td>
</tr>
<tr>
<td>medium</td>
<td>$\mathcal{L}(90, 110)$</td>
<td>100 0.5</td>
</tr>
<tr>
<td>high</td>
<td>$\mathcal{L}(140, 160)$</td>
<td>150 0.75</td>
</tr>
</tbody>
</table>

Therefore, we just present part of them in this part.

Table 8: The effects of $\tilde{c}$ when the competing intensity are high($E[\tilde{\gamma}]=150$)

<table>
<thead>
<tr>
<th>Structure</th>
<th>$\tilde{c}$</th>
<th>$w^*_c$</th>
<th>$p^*_c$</th>
<th>$E[\pi_m]$</th>
<th>$m^*_c$</th>
<th>$p^*_r$</th>
<th>$E[\pi_r]$</th>
<th>$E[\pi_t]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS</td>
<td>$\mathcal{L}(10, 10)$</td>
<td>45.1024</td>
<td>48.2310</td>
<td>106547.93</td>
<td>10.7396</td>
<td>55.8420</td>
<td>13110.85</td>
<td>119658.78</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{L}(9, 11)$</td>
<td>45.2024</td>
<td>48.3310</td>
<td>108006.27</td>
<td>10.7271</td>
<td>55.9295</td>
<td>13088.18</td>
<td>121094.45</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{L}(8, 12)$</td>
<td>45.3024</td>
<td>48.4310</td>
<td>109466.47</td>
<td>10.7146</td>
<td>56.0170</td>
<td>13065.58</td>
<td>122532.05</td>
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<tr>
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<td>$\mathcal{L}(7, 13)$</td>
<td>45.4024</td>
<td>48.5310</td>
<td>110928.56</td>
<td>10.7021</td>
<td>56.1045</td>
<td>13043.04</td>
<td>123971.59</td>
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<tr>
<td>VN</td>
<td>$\mathcal{L}(10, 10)$</td>
<td>41.9649</td>
<td>48.2310</td>
<td>105563.54</td>
<td>12.3083</td>
<td>54.2732</td>
<td>16556.22</td>
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<td>$\mathcal{L}(9, 11)$</td>
<td>42.0732</td>
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<td>12.2917</td>
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<td>$\mathcal{L}(8, 12)$</td>
<td>42.1815</td>
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<td>$\mathcal{L}(7, 13)$</td>
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<td>106569.11</td>
<td>15.3708</td>
<td>56.1045</td>
<td>17402.48</td>
<td>123971.59</td>
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</tbody>
</table>

In the similar way, we can conduct more experiments about the influence of the other parameters’ uncertain degrees on the pricing decisions.
6. Conclusion

In this paper, we considered a supply chain pricing decision problem in which the manufacturer sells to a retailer as well as to consumers through a company store (e.g., online shop and direct-sale store) directly. The demand is characterized as a price-dependent and channel-dependent (offline shop vs. online shop) uncertain function. Meanwhile, manufacturing costs and sale costs are both uncertain variables. In addition, we employed Stackelberg game model and Nash game model to derive how the participants should make their optimal pricing decisions in different power structures. Numerical experiments illustrated the effectiveness of the proposed models.

This paper also examined the impacts of the power structures and uncertain degrees of the parameters on the equilibrium prices and expected profits. It is found that consumers can obtain lower prices when facing uncertain environment. Moreover, the results showed that the channel member with dominance can gain more profits while the whole supply chain gain most profit when there is no dominance in the system. We also concluded that when the uncertain degrees of the some channel members’ costs ($\tilde{c}$, $\tilde{s}_m$ for the manufacturer and $\tilde{s}_r$ for the retailer) increase and the ones can charge a higher prices and gain more profits.

The future researches can focus on some more complicated pricing problems in which randomness and uncertainty might co-exist. Moreover, as service competition has become increasingly important in the real world, and this paper considered the price competition only, taking price and service into consideration simultaneously is another extension of this paper. Besides, this paper assumed that all the channel members are risk neutral, but the research can be more applicable if the channel members’ risk-sensitive behaviors are considered.

Acknowledgments

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References


