Standby Redundancy Optimization Problems with Uncertain Lifetimes and Uncertain Costs

Yufu Ning\textsuperscript{1,2}, Xiumei Chen\textsuperscript{1,2*}, Xiao Wang\textsuperscript{1,2}

\textsuperscript{1}School of Information Engineering, Shandong Youth University of Political Science, Jinan 250103, China
\textsuperscript{2}Key Laboratory of Information Security and Intelligent Control in Universities of Shandong, Jinan 250103, China
cxm512@163.com

Abstract

Redundancy optimization problem with cost constraints is a typical problem in reliability engineering. This study investigates standby redundancy optimization problem with uncertain lifetimes and uncertain costs. Three models for optimizing different performances of a standby system are formulated. In these models, maximizing the reliability measure, maximizing the system lifetime and minimizing the total cost with some constraints are considered as the goals. Finally, a numerical example is presented to illustrate the models.

Keywords: Standby redundancy optimization, reliability, uncertainty theory

1 Introduction

Reliability measures the ability of a system performing its intended functions. It is one of the most critical performance measures of today’s complex systems, such as transportation systems, power systems, communication systems and aircraft systems, and it has been emphasized more and more by the academia, the industry and the government.

Reliability engineering emerged as a separate engineering discipline during 1950s in the United States [16, 18]. The area of reliability engineering focuses on analyzing the reliability of a component or a system and enhancing the reliability through measures in its design, manufacturing and operation. Enhancing system reliability through optimal design is one of the major topics covered by reliability engineering. Component redundancy is an efficient way to enhance system reliability. Generally speaking, two underlying component redundancy techniques are considered. The first one is parallel redundancy where all redundant elements are in parallel and working simultaneously. And parallel redundancy is often employed when the system is required to work for a long period of time without interruption. The other is standby redundancy in which one of the elements begins to work only when the active one failed. And standby redundancy is usually employed when the replacement between elements takes a negligible amount of time which does not cause failure of the system. Determining the number of redundant elements for improving the system performances under some constraints including cost constraint is well known as the redundancy optimization problem. Abundant literatures described redundancy optimization models such as Coit and Smith [1], Coit [2], Zhao [25] and Mostafa [3].

Usually, in a redundancy optimization problem, the lifetimes of elements or systems have been assumed to be random variables and system performances such as system reliability were modeled by probability measure. Although this assumption has been widely used and in many cases it accorded with the facts, it is also not suitable in many situations such as space shuttle system. Due to the complexity of some systems and imprecision of data, the estimations of probability distributions of lifetimes of elements or systems are almost impossible. In this case, possibility theory introduced by Zadeh [23] in 1978 was employed to characterize the
elements lifetimes by fuzzy variables. From then on, many researchers had widely studied fuzzy redundancy optimization problems in the fuzzy theoretic perspective [4, 19, 17, 26]. But Zedah’s fuzzy theory has been challenged by many researchers. Liu [14] claimed that fuzzy set theory was not selfconsistent in mathematics and might lead to wrong results in practice.

With the development of science and technology, various new elements emerge in an endless stream. When encountering the cases when there is not enough information to estimate the probability distribution of lifetimes of new elements or systems, it is reasonable to rely on the belief degrees of the domain experts. Different experts may produce different belief degrees. Liu [14] showed that human beings usually estimate a much wider range of values than the object actually takes. In order to deal with the belief degree mathematically, an uncertainty theory was founded by Liu [6] in 2007 and refined by Liu [7] in 2010 based on normality axiom, duality axiom, subadditivity axiom and product axiom. A concept of uncertain variable was defined by Liu [6] as a measurable function from an uncertainty space to the set of real numbers. Meanwhile, a series of concepts such as uncertainty distribution, independence, expected value, variance and entropy, etc, were also proposed to describe an uncertain variable. Nowadays, there have been vigorous developments in uncertainty theory such as uncertain programming [8], [9], uncertain differential equation [22], uncertain finance [13], and so on.

Uncertain reliability analysis was proposed by Liu [11] in 2010 in which the lifetimes of elements or system were considered as uncertain variables and the reliability index was defined. After that, Wang [20] used uncertainty theory to analyze structural reliability and discussed structural reliability index as the uncertain measure of the event. Zeng et al. [24] defined some new reliability metrics based on uncertainty theory to characterize the systems. Liu [15] analyzed the reliability of redundant system, including cold redundant system and warm redundant system. Wen [21] proposed a method of reliability analysis based on chance theory which is a generalization of both probability theory and uncertainty theory.

In this paper, we further study standby redundancy optimization problems, and focus on three models under uncertainty environment where not only lifetimes of elements but also the costs of elements are considered as uncertain variables. Two of them are with cost constraint to maximize the reliability and the system lifetime, respectively. The other is to minimize the expected value of total cost with reliability constraint. The rest of this paper is organized as follows. In Section 2 some basic concepts and theorems of uncertainty theory are introduced. In Section 3 a system reliability measure is introduced, and standby redundancy optimization problem is displayed in Section 4. In Section 5, three models are presented and their equivalences are obtained by uncertainty theory. After that, numerical experiments to show the effectiveness of the proposed models are given in Section 6. At last, some remarks are made in Section 7.

2 Preliminaries

In this section, we will introduce some basic concepts and theorems about uncertain variable, such as uncertainty distribution, expected value and operational law.

**Definition 2.1** (Liu [6]) Let $\mathcal{L}$ be a $\sigma$-algebra on a nonempty set $\Gamma$. A set function $\mathcal{M}$ is called an *uncertain measure* if it satisfies the following axioms:

Axiom 1. (*Normality Axiom*) $\mathcal{M}\{\Gamma\} = 1$;

Axiom 2. (*Duality Axiom*) $\mathcal{M}\{A\} + \mathcal{M}\{A^c\} = 1$ for any $A \in \mathcal{L}$;

Axiom 3. (*Subadditivity Axiom*) For every countable sequence of $\{A_i\} \in \mathcal{L}$, we have

$$\mathcal{M}\left(\bigcup_{i=1}^{\infty} A_i\right) \leq \sum_{i=1}^{\infty} \mathcal{M}\{A_i\}.$$
The triplet \((\Gamma, \mathcal{L}, M)\) is called an *uncertainty space*, and each element \(\Lambda\) in \(\mathcal{L}\) is called an *event*. In addition, in order to obtain an uncertain measure of compound events, Liu \[10\] defined the product uncertain measure on the product \(\sigma\)-algebra \(\mathcal{L}\) as follows.

**Axiom 4. (Product Axiom)** Let \((\Gamma_k, \mathcal{L}_k, M_k)\) be uncertainty spaces for \(k = 1, 2, \ldots\). The product uncertain measure \(M\) is an uncertain measure satisfying

\[
M \left( \prod_{k=1}^{\infty} \Lambda_k \right) = \bigwedge_{k=1}^{\infty} M_k \{ \Lambda_k \}
\]

where \(\Lambda_k\) are arbitrarily chosen events from \(\mathcal{L}_k\) for \(k = 1, 2, \ldots\), respectively.

In order to model uncertain quantity, Liu \[6\] defined the uncertain variable which is a measurable function from an uncertainty space \((\Gamma, \mathcal{L}, M)\) to the set of real numbers and satisfies the measurable condition.

**Definition 2.2** (Liu \[6\]) The *uncertainty distribution* \(\Phi\) of an uncertain variable \(\xi\) is defined by

\[
\Phi(x) = M \{ \xi \leq x \}, \quad \forall x \in \mathbb{R}.
\]

**Definition 2.3** (Liu \[7\]) An uncertainty distribution \(\Phi(x)\) is said to be *regular* if it is a continuous and strictly increasing function with respect to \(x\) at which \(0 < \Phi(x) < 1\), and

\[
\lim_{x \to -\infty} \Phi(x) = 0, \quad \lim_{x \to +\infty} \Phi(x) = 1.
\]

In addition, the inverse function \(\Phi^{-1}(\alpha)\) is called the *inverse uncertainty distribution* of \(\xi\).

An uncertain variable \(\xi\) is said to be linear if it has a linear uncertainty distribution

\[
\Phi(x) = \begin{cases} 
0, & \text{if } x < a \\
(x-a)/(b-a), & \text{if } a \leq x \leq b \\
1, & \text{if } x > b
\end{cases}
\]

which is denoted by \(\xi \sim \mathcal{L}(a,b)\). Apparently, the linear uncertain variable \(\xi\) is regular, and has an inverse uncertainty distribution

\[
\Phi^{-1}(\alpha) = \alpha(b-a) + a.
\]

An uncertain variable \(\xi\) is said to be normal if it has a normal uncertainty distribution

\[
\Phi(x) = \left(1 + \exp \left( \frac{\pi(e-x)}{\sqrt{3}\sigma} \right) \right)^{-1}, \quad x \in \mathbb{R}
\]

denoted by \(\xi \sim \mathcal{N}(e,\sigma)\) where \(e\) and \(\sigma\) are real numbers with \(\sigma > 0\). The normal uncertain variable is regular, and the inverse uncertainty distribution is

\[
\Phi^{-1}(\alpha) = e + \frac{\sigma\sqrt{3}}{\pi} \ln \frac{\alpha}{1-\alpha}.
\]

**Definition 2.4** (Liu \[10\]) The uncertain variables \(\xi_1, \xi_2, \ldots, \xi_n\) are said to be *independent* if

\[
M \left\{ \bigcap_{i=1}^{n} (\xi_i \in B_i) \right\} = \bigwedge_{i=1}^{n} M \{ \xi_i \in B_i \}
\]

for any Borel sets \(B_1, B_2, \ldots, B_n\) of real numbers.
Definition 2.5 (Liu [6]) Let $\xi$ be an uncertain variable. The expected value of $\xi$ is defined by

$$E[\xi] = \int_0^{+\infty} M(\xi \geq r)dr - \int_{-\infty}^0 M(\xi \leq r)dr$$

provided that at least one of the above two integrals is finite.

For an uncertain variable $\xi$ with an uncertainty distribution $\Phi$, we have

$$E[\xi] = \int_0^{+\infty} (1 - \Phi(r)) dr - \int_{-\infty}^0 \Phi(r) dr.$$

If the inverse uncertainty distribution $\Phi^{-1}$ exists, then

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha) d\alpha.$$

Theorem 2.1 (Liu [7]) Assume $\xi_1, \xi_2, \cdots, \xi_n$ are independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. If the function $f(x_1, x_2, \cdots, x_n)$ is strictly increasing with respect to $x_1, x_2, \cdots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \cdots, x_n$, then $\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$ has an inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = f (\Phi^{-1}_1(\alpha), \cdots, \Phi^{-1}_m(\alpha), \Phi^{-1}_{m+1}(1 - \alpha), \cdots, \Phi^{-1}_n(1 - \alpha)).$$

Based on this result and the formula $E[\xi] = \int_0^1 \Phi^{-1}(\alpha) d\alpha$, Liu [12] proved the linearity of expected value operator.

Theorem 2.2 Let $\xi$ and $\eta$ be independent uncertain variables with finite expected values. Then for any real numbers $a$ and $b$, we have

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta].$$

Theorem 2.3 (Liu [12]) Assume $\xi_1, \xi_2, \cdots, \xi_n$ are independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. Then the sum $\xi_1 + \xi_2 + \cdots + \xi_n$, the minimum $\xi_1 \land \xi_2 \land \cdots \land \xi_n$ and the maximum $\xi_1 \lor \xi_2 \lor \cdots \lor \xi_n$ have uncertainty distributions $\sup_{x_1 + x_2 + \cdots + x_n = x} \Phi_i(x)$, $\Phi_1(x) \lor \Phi_2(x) \lor \cdots \lor \Phi_n(x)$, and $\Phi_1(x) \land \Phi_2(x) \land \cdots \land \Phi_n(x)$, respectively.

3 Uncertainty Reliability

The lifetime of an element or a system is the length of time interval from the initial activation to its failure. Since the lifetime cannot be exactly predicted, it is considered as an uncertain variable in this paper. Liu [11] first introduced the definition of reliability index as follows.

Definition 3.1 (Liu [11]) Assume a system contains uncertain variables $\xi_1, \xi_2, \cdots, \xi_n$, and there is a function $R$ such that the system is working if and only if $R(\xi_1, \xi_2, \cdots, \xi_n) > 0$. Then the reliability index is

$$\text{Reliability} = M\{R(\xi_1, \xi_2, \cdots, \xi_n) > 0\}.$$

Example 3.1 Consider a series system in which there are $n$ elements whose lifetimes are independent uncertain variables $\xi_1, \xi_2, \cdots, \xi_n$ with uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. Such a system fails if any one element does not work. Thus by Theorem 2.3 the system lifetime

$$\xi = \xi_1 \land \xi_2 \land \cdots \land \xi_n$$
is an uncertain variable with uncertainty distribution
\[ \Psi(x) = \Phi_1(x) \lor \Phi_2(x) \lor \cdots \lor \Phi_n(x). \]

If the system being working is understood as the case that the system does not fail before time \( T \), then we have
\[ R(\xi_1, \xi_2, \cdots, \xi_n) = \xi_1 \land \xi_2 \land \cdots \land \xi_n - T. \]

Hence the reliability of the series system is
\[ \text{Reliability} = \mathcal{M}\{\xi_1 \land \xi_2 \land \cdots \land \xi_n > T\} = 1 - \Phi_1(T) \lor \Phi_2(T) \lor \cdots \lor \Phi_n(T). \]

**Example 3.2** Consider a parallel system in which there are \( n \) elements whose lifetimes are independent uncertain variables \( \xi_1, \xi_2, \cdots, \xi_n \) with uncertainty distributions \( \Phi_1, \Phi_2, \cdots, \Phi_n \), respectively. Such a system fails if all elements do not work. Thus by Theorem 2.3 the system lifetime
\[ \xi = \xi_1 \lor \xi_2 \lor \cdots \lor \xi_n \]
is an uncertain variable with uncertainty distribution
\[ \Psi(x) = \Phi_1(x) \land \Phi_2(x) \land \cdots \land \Phi_n(x). \]

If the system being working is understood as the case that the system does not fail before time \( T \), then we have
\[ R(\xi_1, \xi_2, \cdots, \xi_n) = \xi_1 \lor \xi_2 \lor \cdots \lor \xi_n - T. \]

Hence the reliability of the parallel system is
\[ \text{Reliability} = \mathcal{M}\{\xi_1 \lor \xi_2 \lor \cdots \lor \xi_n > T\} = 1 - \Phi_1(T) \land \Phi_2(T) \land \cdots \land \Phi_n(T). \]

**Example 3.3** Consider a standby system in which there are \( n \) elements with \( n-1 \) redundant elements whose lifetimes are independent uncertain variables \( \xi_1, \xi_2, \cdots, \xi_n \) with uncertainty distributions \( \Phi_1, \Phi_2, \cdots, \Phi_n \), respectively. For this system, only one element is active and one of the redundant elements begins to work only when the active element fails. Thus by Theorem 2.3 the system lifetime
\[ \xi = \xi_1 + \xi_2 + \cdots + \xi_n \]
is an uncertain variable with uncertainty distribution
\[
\sup_{x_1 + x_2 + \cdots + x_n = x} \min_{1 \leq i \leq n} \Phi_i(x_i).
\]
If the system being working is understood as the case that the system does not fail before time \(T\), then we have
\[
R(\xi_1, \xi_2, \cdots, \xi_n) = \xi_1 + \xi_2 + \cdots + \xi_n - T.
\]
Hence the reliability of the standby system is
\[
Reliability = \mathcal{M}\{\xi_1 + \xi_2 + \cdots + \xi_n > T\} = 1 - \sup_{x_1 + x_2 + \cdots + x_n = T} \min_{1 \leq i \leq n} \Phi_i(x_i).
\]

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{standby_system_1_component.png}
\caption{A Standby System with One Component}
\end{figure}

4 Standby Redundancy Optimization Problems

In this section, we focus on a standby system. Assume that the standby system is composed of \(n\) components, and the \(i\)th component is composed of \(x_i - 1\) redundant elements (for example, see Figure 4) for \(i = 1, 2, \cdots, n\), respectively. In addition, the following hypotheses are considered.

H1. For the \(i\)th component, \((1 \leq i \leq n)\), there is one element available.
H2. Both the components and the system only take two possible states: completely working and totally failed.
H3. There is no element or system repair or preventive maintenance.
H4. The switches of the standby system are assumed to be perfected.
H5. The elements do not fail before they are put into operation.
H6. Lifetimes of elements are uncertain variables.
H7. All elements are independent.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{standby_system_n_components.png}
\caption{A Standby System with \(n\) Components}
\end{figure}

We utilize the following notations. Let \(n\) represent the number of components and \(x_i\) denote the numbers of elements in the \(i\)th component for \(i = 1, 2, \cdots, n\), respectively. The redundant optimization problem is to find the optimal value of \(x = (x_1, x_2, \cdots, x_n)\) so that some system performances are optimized.
Let $\xi_{ij}$ indicate the uncertain lifetime for the $j$th element in $i$th component with uncertainty distribution $\Phi_{ij}$, and $T_i(x, \xi)$ denote the lifetime of $i$th component for $j = 1, 2, \ldots, x_i, i = 1, 2, \ldots, n$, respectively, where uncertain lifetime vector $\xi = (\xi_{11}, \xi_{12}, \ldots, \xi_{x_1}, \xi_{21}, \xi_{22}, \ldots, \xi_{x_2}, \ldots, \xi_{n1}, \xi_{n2}, \ldots, \xi_{nx_n})$. Let $T(x, \xi)$ be the lifetime of the system. Then $T(x, \xi)$ is determined fully by $T_i(x, \xi)$, in further, determined by $\xi$. For example, in the standby system with $n$ components in Figure 4, $T_i(x, \xi) = \sum_{1 \leq j \leq x_i} \xi_{ij}$ and $T(x, \xi) = \min_{1 \leq i \leq n} T_i(x, \xi)$. Obviously, both $T_i(x, \xi)$ and $T(x, \xi)$ are uncertain variables.

5 Uncertain Redundancy Optimization Models

The redundancy optimization problem is to determine the number of redundant elements for enhancing the system performances under certain constraints. In practical situations involving reliability optimization, there often exist conflicting goals such as maximizing the system reliability, maximizing the system lifetime and minimizing the total cost. More attention is paid to the reliability of a system with the cost constraint, and it is indeed an important goal in redundancy optimization problem. For different decision makers, different system performances may be emphasized. If more caring for the cost of elements it is suitable to minimize the costs with reliability constraint. Since the cost of the elements are time-varying, so it is reasonable to consider the costs of the elements as uncertain variables. Since the costs of the elements are uncertain variables, it is meaningless to minimize an uncertain variable unless a criterion of ranking uncertain variables is offered. An alternative is to minimize the expected value of the costs. Let $\eta_i$ be independent uncertain variables representing the cost of the $i$th component for $i = 1, 2, \ldots, n$, respectively. And we give the following three models considering different goals. If we maximize the uncertain measure $\mathcal{M}\{T(x, \xi) > t_0\}$, i.e., the reliability of this system at $t_0$, we give the following model with uncertain cost constraint:

$$
\begin{align*}
\max \mathcal{M}\{T(x, \xi) > t_0\} \\
\text{subject to:} \\
E[\sum_{1 \leq i \leq n} x_i \eta_i] \leq c_0 \\
x \geq 1, \text{integer vector.}
\end{align*}
$$

(1)

If we maximize the lifetime of the system, i.e., the $\alpha_0$-optimistic value of lifetime $T(x, \xi)$, we give the following model:

$$
\begin{align*}
\max x_T \\
\text{subject to:} \\
\mathcal{M}\{T(x, \xi) > t_x\} \geq \alpha_0 \\
E[\sum_{1 \leq i \leq n} x_i \eta_i] \leq c_0 \\
x \geq 1, \text{integer vector}
\end{align*}
$$

(2)

where $t_x$ represents the time related to $x$. The results of model (1) and (2) can tell us the maximum reliability of system and the longest time that the system may work with cost constraint, respectively. In the following, we establish a model which more emphasizes on the cost of the elements. And the constraint is that the reliability of the system at $t_0$ surpasses $\alpha_0$. Then we have the model as follows:
\[
\begin{align*}
\min_E \sum_{1 \leq i \leq n} x_i \eta_i \\
\text{subject to:} \\
\mathcal{M}\{T(x, \xi) > t_0\} \geq \alpha_0 \\
x \geq 1, \text{ integer vector.}
\end{align*}
\]

(3)

In the following, we analyse the above expressions and transfer them into concrete. By using the linearity of expected value, we have \(E[\sum_{1 \leq i \leq n} x_i \eta_i] = \sum_{1 \leq i \leq n} x_i E[\eta_i]\). Let \(\Phi(x, t)\) be the regular distribution of \(T(x, \xi)\) and \(\Phi_i(x, t)\) be the regular distribution of the lifetime of \(i\)th component \(T_i(x, \xi)\) for \(i = 1, 2, \cdots, n\), respectively. Note that \(T(x, \xi) = \min_{1 \leq i \leq n} T_i(x, \xi) = \min_{1 \leq i \leq n} \sum_{1 \leq j \leq x_i} \xi_{ij}\). It follows from Theorem 2.1 that the inverse uncertainty distribution of \(T(x, \xi)\) is strictly increasing,

\[
\Phi_i(x, t) = \max_{1 \leq i \leq n} \Phi_i \left( \frac{t}{x_i} \right).
\]

By the definition of uncertainty distribution and Duality Axiom, \(\mathcal{M}\{T(x, \xi) > t_0\} = 1 - \Phi(x, t_0) = 1 - \max_{1 \leq i \leq n} \Phi_i \left( \frac{t_0}{x_i} \right)\). Hence Model (1) is equivalent to

\[
\begin{align*}
\max & \left( 1 - \max_{1 \leq i \leq n} \Phi_i \left( \frac{t_0}{x_i} \right) \right) \\
\text{subject to:} \\
\sum_{1 \leq i \leq n} x_i E[\eta_i] \leq c_0 \\
x \geq 1, \text{ integer vector.}
\end{align*}
\]

(4)

It follows from Theorem 2.1 that the inverse uncertainty distribution of \(T(x, \xi)\) is

\[
\Phi^{-1}(\alpha) = \min_{1 \leq i \leq n} \sum_{1 \leq j \leq x_i} \Phi^{-1}_{ij} (\alpha) = \min_{1 \leq i \leq n} x_i \Phi^{-1}_i (\alpha)
\]

where \(\Phi^{-1}_{ij}(\alpha)\) is the inverse uncertainty distribution of \(j\)th element in \(i\)th component and \(\Phi^{-1}_i(\alpha)\) is the inverse uncertainty distribution of \(i\)th component for \(j = 1, 2, \cdots, x_i, i = 1, 2, \cdots, n\), respectively. Hence, \(\mathcal{M}\{T(x, \xi) > t_x\} \geq \alpha_0\) is equivalent to \(\Phi(x, t_x) \leq 1 - \alpha_0\) which is also equivalent to \(t_x \leq \Phi^{-1}(1 - \alpha_0)\). Since \(\Phi(x, t_x)\) is strictly increasing, \(t_x\) can reach its maximum and \(t_x = \Phi^{-1}(1 - \alpha_0)\). Hence model (2) is equivalent to

\[
\begin{align*}
\max & \Phi^{-1}(1 - \alpha_0) \\
\text{subject to:} \\
\sum_{1 \leq i \leq n} x_i E[\eta_i] \leq c_0 \\
x \geq 1, \text{ integer vector.}
\end{align*}
\]

(5)

And model (3) is equivalent to

\[
\begin{align*}
\min & x_i E[\eta_i] \\
\text{subject to:} \\
\Phi^{-1}(1 - \alpha_0) \geq t_0 \\
x \geq 1, \text{ integer vector.}
\end{align*}
\]

(6)
Note that model (4), model (5) and model (6) are crisp nonlinear integer programming models. So we can solve the models by the integer programming software Lingo 10.0.

6 Experimental Illustration

In this section, we will provide an example to show the effectiveness of the models. Considering a standby system with five elements, we display the detailed database in Table 1.

Table 1: Elements Lifetimes and Costs

<table>
<thead>
<tr>
<th>Component</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element Lifetime</td>
<td>N(12,3)</td>
<td>L(12,16)</td>
<td>L(11,15)</td>
<td>N(10,2)</td>
<td>L(14,18)</td>
</tr>
<tr>
<td>Element Cost</td>
<td>N(6,1)</td>
<td>N(7,0.5)</td>
<td>L(8,10)</td>
<td>L(10,12)</td>
<td>N(9,2)</td>
</tr>
</tbody>
</table>

If we take $\alpha_0$ is 0.9, $c_0$ is 400, $t_0$ is 100, then model (4) is

$$
\begin{align*}
\max & (1 - \max_{1 \leq i \leq 5} \Phi_i(\frac{100}{x_i})) \\
\text{subject to:} & \\
6x_1 + 7x_2 + 9x_3 + 11x_4 + 9x_5 & \leq 400 \\
x & \geq 1, \text{integer vector}
\end{align*}
$$

which is a crisp integer programming. A run of Lingo 10.0 shows that the optimal solution is $x = (11, 8, 9, 12, 7)$ and the corresponding maximum reliability of system is 0.8193, which tells us that in order to maximize the system reliability, we should take 10 redundant elements in the first component, 7 redundant elements in the second component, 8 redundant elements in the third component, 11 redundant elements in the forth component and 6 redundant elements in the fifth component.

And the equivalent model (5) is

$$
\begin{align*}
\max & \left( \min_{1 \leq i \leq 5} x_i \Phi_i^{-1}(0.1) \right) \\
\text{subject to:} & \\
6x_1 + 7x_2 + 9x_3 + 11x_4 + 9x_5 & \geq 100 \\
x & \geq 1, \text{integer vector}
\end{align*}
$$

which is a crisp integer programming. A run of Lingo 10.0 shows that the optimal solution is $x = (11, 8, 8, 13, 7)$ and the corresponding maximum lifetime is 91.2, which implies to us that in order to maximize the system lifetime, we should take 10 redundant elements in the first component, 7 redundant elements in the second component, 7 redundant elements in the third component, 12 redundant elements in the forth component and 6 redundant elements in the fifth component.

Similarly, model (6) is

$$
\begin{align*}
\min & \{6x_1 + 7x_2 + 9x_3 + 11x_4 + 9x_5\} \\
\text{subject to:} & \\
\min_{1 \leq i \leq 5} \{x_i \Phi_i^{-1}(0.1)\} & \geq 100 \\
x & \geq 1, \text{integer vector}
\end{align*}
$$

which also is a crisp integer programming. A run of Lingo 10.0 shows that the optimal solution is $x = (12, 9, 9, 14, 7)$ and the corresponding minimum cost is 433, which means that in order to minimize the total cost of the system, we should take 11 redundant elements in the first component, 8 redundant elements in the second component, 8 redundant elements in the third component, 13 redundant elements in the forth component and 6 redundant elements in the fifth component.
7 Conclusions

This paper concerns about standby redundancy optimization problems, where the lifetimes and costs of elements were considered as uncertain variables. Under the framework of uncertainty theory, we established three models to optimize the systems’ performances under some constraints. The models are equivalent to nonlinear integer programming models. Finally, a numerical example was given to illustrate the effectiveness of the models. By running software Lingo 10.0 we can obtain the optimization solution.

Acknowledgements

This work is supported by Natural Science Foundation of Shandong Province (ZR2014GL002).

References


