Elliptic entropy of uncertain random variables

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Abstract: Entropies of uncertain random variables are introduced to provide some kinds of quantitative measurement of the uncertainty. In chance theory, a concept of entropy for uncertain random variables has been defined by using logarithm. However, such an entropy fails to measure the uncertain degree of some uncertain random variables. For solving this problem, this paper firstly define the elliptic entropy to characterize the uncertainty of uncertain random variables. Secondly, some properties of elliptic entropy are discussed. Finally, triangular and radical entropies of uncertain random variables are investigated.

Keywords: Chance theory; uncertain random variable; entropy; elliptic entropy

1 Introduction

Entropy, as a measure of information quantity, was first proposed by Shannon [22] for a discrete random variable in the form of logarithm in 1949. Then, Jaynes [10] proposed a maximum entropy principle, which is to choose the one with the maximum entropy in practice among the random variables with the same expected value and variance.

To deal with humans uncertainty, Liu [14] founded an uncertainty theory based on an uncertain measure which satisfies normality, duality, subadditivity and product axioms. Liu [15] proposed a concept of entropy for uncertain variables in the form of logarithm function. Then, Dai and Chen [6] gave a formula to calculate the entropy of a function of uncertain variables, and Chen and Dai [4] proposed the maximum entropy principle for uncertain variables. After that, the properties of entropy for uncertain variables were further studied by Chen et al. [5], Ning et al. [21], Tang and Gao [24], and Yao et al. [25].

As the system becomes more complex, the uncertainty and randomness are required to be considered simultaneously in a system. In order to describe this phenomenon, chance theory was pioneered by Liu [16] via giving the concepts of uncertain random variable and chance measure. Some basic concepts of chance theory such as chance distribution, expected value, and variance were also proposed by Liu [16]. As an important contribution to chance theory, Liu [16] presented an operational law of uncertain random variables. After that, chance theory was developed steadily and applied widely.

In 2015, Sheng et al. [23] provided a definition of entropy to characterize the uncertainty of un-
certain random variable. However, the concept of entropy for uncertain random variables proposed by Sheng et al. [23] has a drawback. That is, it cannot answer the following question: How much of entropy of uncertain random variable associated to uncertain variable? For solving this problem, Ahmadzade et al. [1] employ a new approach to define the concept of entropy for uncertain random variables. After that, Ahmadzade et al. [2] provided the quadratic entropy of uncertain random variables, and verified its properties such as translation invariance and positive linearity.

In order to explore diverse definitions of entropy for uncertain random variables and provide more mathematical tools to engineers, this paper provides a new type of entropy called elliptic entropy and presents some properties of the elliptic entropy. The remainder of this paper is organized as follows. Section 2 presents some basic concepts and theorems about the uncertainty theory and chance theory, respectively. In Section 3, a definition of elliptic entropy of uncertain random variable is proposed and some properties are presented. As supplements, other types of entropies are studied in Section 4. At the end of the paper, a brief summary is presented.

2 Preliminary

Liu [16] proposed the chance theory, which is a mathematical methodology for modeling complex systems with both uncertainty and randomness. Ahmadzade et al. [2] obtained several formulas to calculate the moments of an uncertain random variable. After that, Ahmadzade et al. [3] studied some properties of uncertain random sequences. Gao and Sheng [7] studied the law of large numbers of uncertain random variables. Liu and Yao [20] proposed an uncertain random logic to deal with the uncertain random knowledge. Besides, chance theory was applied into many fields, such as uncertain random programming (Liu [17], Ke et al. [13], Ke et al. [12]), uncertain random system (Gao et al. [8], Gao and Yao [9]), uncertain random risk analysis (Liu and Ralescu [18], Liu and Ralescu [19]), uncertain random network (Ke et al. [11]).

Assume that \((\Gamma, \mathcal{L}, M)\) is an uncertainty space, and \((\Omega, \mathcal{A}, \Pr)\) is a probability space. The product \((\Gamma, \mathcal{L}, M) \times (\Omega, \mathcal{A}, \Pr)\) is called a chance space. Each element \(\Theta \in \mathcal{L} \times \mathcal{A}\) is called an event in the chance space. The chance measure of event \(\Theta\) is defined by Liu [16] as

\[
\text{Ch} \{\Theta\} = \int_0^1 \Pr \{\omega \in \Omega \mid M \{\gamma \in \Gamma \mid (\gamma, \omega) \in \Theta\} \geq x\} \, dx.
\]

Liu [17] provided the following operation law to determine the chance distribution of uncertain random variable. Assume that \(\eta_1, \eta_2, \ldots, \eta_m\) are independent random variables with probability distributions \(\Psi_1, \Psi_2, \ldots, \Psi_m\), respectively, and assume that \(\tau_1, \tau_2, \ldots, \tau_n\) are uncertain variables (not necessarily independent). Then the uncertain random variable

\[
\xi = f(\eta_1, \eta_2, \ldots, \eta_m, \tau_1, \tau_2, \ldots, \tau_n)
\]
has a chance distribution

$$\Phi(x) = \int_{\mathbb{R}^m} F(x; y_1, y_2, \cdots, y_m) d\Psi_1(y_1) \cdots d\Psi_m(y_m),$$

where \(F(x; y_1, y_2, \cdots, y_m)\) is the uncertainty distribution of \(f(y_1, y_2, \cdots, y_m, \tau_1, \tau_2, \cdots, \tau_n)\) for any real numbers \(y_1, y_2, \cdots, y_m\).

### 3 Elliptic Entropy of Uncertain Random Variable

In chance theory, Sheng et al. [23] put forward the concept of entropy for uncertain random variables in 2015. However, the concept of entropy for uncertain random variables proposed by Sheng et al. [23] has a drawback. That is, it cannot answer the following question: How much of entropy of uncertain random variable associated to uncertain variable? For solving this problem, Ahmadzade et al. [1] used a new approach to define the concept of entropy for uncertain random variables as follows.

**Definition 3.1 (Ahmadzade et al. [1])** Suppose that \(\eta_1, \eta_2, \cdots, \eta_m\) are independent random variables with probability distributions \(\Psi_1, \Psi_2, \cdots, \Psi_m\), respectively, and \(\tau_1, \tau_2, \cdots, \tau_n\) are uncertain variables. Triangular entropy of uncertain random variable \(\xi = f(\eta_1, \cdots, \eta_m, \tau_1, \cdots, \tau_n)\) is defined as following

$$H[\xi] = \int_{\mathbb{R}^m} \int_{-\infty}^{+\infty} S(F(x, y_1, \cdots, y_m))dx d\Psi_1(y_1) \cdots d\Psi_m(y_m),$$

where \(S(t) = -t \ln t - (1-t) \ln(1-t)\) and \(F(x, y_1, \cdots, y_m)\) is the uncertainty distribution of uncertain variable \(f(y_1, \cdots, y_m, \tau_1, \cdots, \tau_n)\) for any real numbers \(y_1, \cdots, y_m\).

Following with Ahmadzade et al. [1], we define the elliptic entropy of uncertain random variable as follows.

**Definition 3.2** Suppose that \(\eta_1, \eta_2, \cdots, \eta_m\) are independent random variables with probability distributions \(\Psi_1, \Psi_2, \cdots, \Psi_m\), respectively, and \(\tau_1, \tau_2, \cdots, \tau_n\) are uncertain variables. Elliptic entropy of uncertain random variable \(\xi = f(\eta_1, \cdots, \eta_m, \tau_1, \cdots, \tau_n)\) is defined as following

$$H[\xi] = \int_{\mathbb{R}^m} \int_{-\infty}^{+\infty} S(F(x, y_1, \cdots, y_m))dx d\Psi_1(y_1) \cdots d\Psi_m(y_m),$$

where \(g(t) = 2k \sqrt{t(1-t)}\) and \(F(x, y_1, \cdots, y_m)\) is the uncertainty distribution of uncertain variable \(f(\eta_1, \cdots, \eta_m, \tau_1, \cdots, \tau_n)\) for any real numbers \(y_1, \cdots, y_m\).

Note that \(2k \sqrt{t(1-t)}\) is the upside of an ellipse and a symmetric function with respect to \(t = 0.5\), and it is strictly increasing in \([0, 0.5]\) and strictly decreasing in \([0.5, 1]\). Moreover, \(k\) is the half axis of ellipse and takes values on the interval \((0, +\infty)\). Besides, ellipse is closely related to elliptic function, elliptic integral and elliptic curve which are widely applied in information sciences. So elliptic entropy can provide an useful quantitative measurement of the uncertainty.
Theorem 3.3 Let \( \eta_1, \eta_2, \ldots, \eta_m \) be independent random variables with probability distributions \( \Psi_1, \Psi_2, \ldots, \Psi_m \), respectively, and let \( \tau_1, \tau_2, \ldots, \tau_n \) be independent uncertain variables. If function \( f \) is a measurable function, then uncertain random variables \( \xi = f(\eta_1, \ldots, \eta_m, \tau_1, \ldots, \tau_n) \) has elliptic entropy
\[
H[\xi] = k \int_{\mathbb{R}^m} \int_{-\infty}^{+\infty} F^{-1}(x, y_1, \ldots, y_m) \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\alpha d\Psi(y_1) \cdots d\Psi(y_m).
\]
Proof. Let \( g(\alpha) = 2k\sqrt{\alpha(1-\alpha)} \). Then \( g(\alpha) \) is a derivable function with \( g'(\alpha) = k \frac{1-2\alpha}{\sqrt{\alpha(1-\alpha)}} \). Since
\[
g(F(x, y_1, \ldots, y_m)) = \int_0^{F(x,y_1,\ldots,y_m)} g'(\alpha) d\alpha,
\]
we have
\[
H[\xi] = \int_{\mathbb{R}^m} \int_{-\infty}^{+\infty} g(F(x, y_1, \ldots, y_m))dx d\Psi(y_1) \cdots d\Psi(y_m)
\]
\[
= \int_{\mathbb{R}^m} \int_{-\infty}^{0} F(x, y_1, \ldots, y_m) g'(\alpha) d\alpha d\Psi(y_1) \cdots d\Psi(y_m)
\]
\[
- \int_{\mathbb{R}^m} \int_{0}^{+\infty} F(x, y_1, \ldots, y_m) g'(\alpha) d\alpha d\Psi(y_1) \cdots d\Psi(y_m).
\]
It follows from Fubini theorem that
\[
H[\xi] = \int_{\mathbb{R}^m} \int_{0}^{F(0,y_1,\ldots,y_m)} g'(\alpha) d\alpha d\Psi(y_1) \cdots d\Psi(y_m)
\]
\[
- \int_{\mathbb{R}^m} \int_{F(0,y_1,\ldots,y_m)}^{+\infty} g'(\alpha) d\alpha d\Psi(y_1) \cdots d\Psi(y_m)
\]
\[
- \int_{\mathbb{R}^m} \int_{0}^{F(0,y_1,\ldots,y_m)} g'(\alpha) d\alpha d\Psi(y_1) \cdots d\Psi(y_m)
\]
\[
- \int_{\mathbb{R}^m} \int_{F(0,y_1,\ldots,y_m)}^{+\infty} g'(\alpha) d\alpha d\Psi(y_1) \cdots d\Psi(y_m)
\]
\[
= k \int_{\mathbb{R}^m} \int_{0}^{F^{-1}(\alpha,y_1,\ldots,y_m)} \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\alpha d\Psi(y_1) \cdots d\Psi(y_m).
\]
Thus the proof is finished. \( \square \)

Theorem 3.4 Let \( \tau \) be an uncertain variable with uncertainty distribution function \( \Phi \) and let \( \eta \) be a random variable with probability distribution function \( \Psi \). If \( \xi = \tau \eta \), then
\[
H[\xi] = H[\tau]E[\eta].
\]
Proof. It is obvious that \( F^{-1}(\alpha, y) = \Phi^{-1}(\alpha)y \), therefore by using Theorem 3.3, we obtain
\[
H[\xi] = k \int_{\mathbb{R}} \int_{0}^{1} \Phi^{-1}(\alpha)y \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\alpha d\Psi(y)
\]
\[
= k \int_{0}^{1} \Phi^{-1}(\alpha) \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\alpha \int_{\mathbb{R}} y d\Psi(y)
\]
\[
= H[\tau]E[\eta].
\]
\( \square \)
**Theorem 3.5** Let \( \tau \) be an uncertain variable with uncertainty distribution function \( \Phi \) and \( \eta \) be a random variable with probability distribution function \( \Psi \). If \( \xi = \eta + \tau \), then

\[
H[\xi] = H[\tau].
\]

Proof. It is obvious that \( F^{-1}(\alpha, y) = \Phi^{-1}(\alpha) + y \), therefore by using Theorem 3.3, we obtain

\[
H[\xi] = k \int_{\mathbb{R}} \int_{0}^{1} (\Phi^{-1}(\alpha) + y) \frac{2\alpha - 1}{\sqrt{\alpha(1 - \alpha)}} d\alpha d\Psi(y)
\]

\[
= k \int_{\mathbb{R}} \int_{0}^{1} \Phi^{-1}(\alpha) \frac{2\alpha - 1}{\sqrt{\alpha(1 - \alpha)}} d\alpha d\Psi(y) + k \int_{\mathbb{R}} \int_{0}^{1} y \frac{2\alpha - 1}{\sqrt{\alpha(1 - \alpha)}} d\alpha d\Psi(y)
\]

\[
= k \int_{0}^{1} \Phi^{-1}(\alpha) \frac{2\alpha - 1}{\sqrt{\alpha(1 - \alpha)}} d\alpha \int_{\mathbb{R}} d\Psi(y) + k \int_{0}^{1} \frac{2\alpha - 1}{\sqrt{\alpha(1 - \alpha)}} d\alpha \int_{\mathbb{R}} y d\Psi(y)
\]

\[
= H[\tau].
\]

Thus the proof is complete. \( \square \)

**Theorem 3.6** Let \( \eta_1, \eta_2, \ldots, \eta_n \) be independent random variables, and let \( \tau_1, \tau_2, \ldots, \tau_n \) be independent uncertain variables. Also, suppose that

\[
\xi_1 = f_1(\eta_1, \tau_1), \xi_2 = f_2(\eta_2, \tau_2), \ldots, \xi_n = f_n(\eta_n, \tau_n).
\]

If \( f(x_1, x_2, \ldots, x_n) \) is strictly increasing with respect to \( x_1, x_2, \ldots, x_m \) and strictly decreasing with respect \( x_{m+1}, x_{m+2}, \ldots, x_n \), then \( \xi = f(\eta_1, \ldots, \eta_m, \tau_1, \ldots, \tau_n) \) has elliptic entropy

\[
H[\xi] = k \int_{\mathbb{R}^m} \int_{0}^{1} f(F_1^{-1}(\alpha, y_1), \ldots, F_{m+1}^{-1}(1-\alpha, y_{m+1}), \ldots, F_n^{-1}(1-\alpha, y_n)) \frac{2\alpha - 1}{\sqrt{\alpha(1 - \alpha)}} d\alpha d\Psi_1(y_1) \cdots d\Psi_n(y_n),
\]

where \( F_i^{-1}(\alpha, y_i) \) is the inverse uncertainty distribution of uncertain variable \( f_i(\tau_i, y_i) \) for any real number \( y_i, i = 1, 2, \ldots, n \).

Proof. It follows from Theorems 2.1 and 3.1 immediately. We omit it. \( \square \)

**Corollary 3.7** Let \( \eta_1, \eta_2, \ldots, \eta_n \) be independent random variables, and \( \tau_1, \tau_2, \ldots, \tau_n \) be independent uncertain variables. Also, suppose that

\[
\xi_1 = f_1(\eta_1, \tau_1), \xi_2 = f_2(\eta_2, \tau_2), \ldots, \xi_n = f_n(\eta_n, \tau_n).
\]

If \( f(x_1, x_2, \ldots, x_n) \) is strictly increasing with respect to \( x_1, x_2, \ldots, x_n \), then \( \xi = f(\eta_1, \eta_2, \ldots, \tau_n) \) has elliptic entropy

\[
H[\xi] = k \int_{\mathbb{R}^m} \int_{0}^{1} f(F_1^{-1}(\alpha, y_1), F_2^{-1}(\alpha, y_2), \ldots, F_n^{-1}(\alpha, y_n)) \frac{2\alpha - 1}{\sqrt{\alpha(1 - \alpha)}} d\alpha d\Psi_1(y_1) \cdots d\Psi_n(y_n).
\]
Corollary 3.8 Let \( \eta_1, \eta_2, \ldots, \eta_n \) be independent random variables, and \( \tau_1, \tau_2, \ldots, \tau_n \) be independent uncertain variables. Also, suppose that
\[
\xi_1 = f_1(\eta_1, \tau_1), \xi_2 = f_2(\eta_2, \tau_2), \ldots, \xi_n = f_n(\eta_n, \tau_n).
\]
If \( f(x_1, x_2, \ldots, x_n) \) is strictly decreasing with respect to \( x_1, x_2, \ldots, x_n \), then \( \xi = f(\eta_1, \eta_2, \ldots, \eta_n) \) has elliptic entropy
\[
H[\xi] = k \int \int_{\mathbb{R}^m} f(F^{-1}_1(1-\alpha, y_1), F^{-1}_2(1-\alpha, y_2), \ldots, F^{-1}_n(1-\alpha, y_n)) \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\Psi_1(y_1) \cdots d\Psi_n(y_n).
\]

Theorem 3.9 Let \( \eta_1 \) and \( \eta_2 \) be random variables with probability distribution functions \( \Psi_1 \) and \( \Psi_2 \), respectively, and \( \tau_1 \) and \( \tau_2 \) be uncertain variables with uncertainty distribution functions \( \Phi_1 \) and \( \Phi_2 \), respectively. If \( \xi_1 = \eta_1 + \tau_1 \) and \( \xi_2 = \eta_2 + \tau_2 \), then
\[
H[\xi_1, \xi_2] = H[\tau_1 \tau_2] + H[\tau_2]E[\eta_1] + H[\tau_1]E[\eta_2].
\]

Proof. It is clear that \( F^{-1}_1(1, y_1) = y_1 + \Phi_1^{-1}(1, \alpha) \) and \( F^{-1}_2(1, y_2) = y_2 + \Phi_2^{-1}(1, \alpha) \). By using Theorem 3.6, we have
\[
H[\xi] = k \int \int_{\mathbb{R}^m} \int_0^1 F^{-1}_1(1, y_1) F^{-1}_2(1, y_2) \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\Psi_1(y_1) d\Psi_2(y_2)
= k \int \int_{\mathbb{R}^m} \int_0^1 \Phi_1^{-1}(1, \alpha) \Phi_2^{-1}(1, \alpha) \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\Psi_1(y_1) d\Psi_2(y_2)
+ k \int \int_{\mathbb{R}^m} \int_0^1 y_1(1, \alpha) \Phi_2^{-1}(1, \alpha) \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\Psi_1(y_1) d\Psi_2(y_2)
+ k \int \int_{\mathbb{R}^m} \int_0^1 y_2(1, \alpha) \Phi_1^{-1}(1, \alpha) \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\Psi_1(y_1) d\Psi_2(y_2)
= k \int_0^1 \Phi_1^{-1}(1, \alpha) \Phi_2^{-1}(1, \alpha) \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\alpha \int_{\mathbb{R}^m} d\Psi_1(y_1) d\Psi_2(y_2)
+ k \int_0^1 (1, \alpha) \Phi_2^{-1}(1, \alpha) \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\alpha \int_{\mathbb{R}^m} y_1 d\Psi_1(y_1) d\Psi_2(y_2)
+ k \int_0^1 (1, \alpha) \Phi_1^{-1}(1, \alpha) \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\alpha \int_{\mathbb{R}^m} y_2 d\Psi_1(y_1) d\Psi_2(y_2)
= H[\tau_1 \tau_2] + H[\tau_2]E[\eta_1] + H[\tau_1]E[\eta_2].
\]

Theorem 3.10 Let \( \eta_1 \) and \( \eta_2 \) be random variables with probability distribution functions \( \Psi_1 \) and \( \Psi_2 \), respectively, and \( \tau_1 \) and \( \tau_2 \) be uncertain variables with uncertainty distribution functions \( \Phi_1 \) and \( \Phi_2 \), respectively. Also suppose that \( \xi_1 = f(\eta_1, \tau_1) \) and \( \xi_2 = f(\eta_2, \tau_2) \). Then for any real numbers \( a \) and \( b \), we have
\[
H[a \xi_1 + b \xi_2] = |a| H[\xi_1] + |b| H[\xi_2].
\]
Suppose that $H[\xi] = |a|H[\xi_1]$. If $a > 0$, then $af(\tau_1, y_1)$ has an inverse uncertainty distribution $F^{-1}(\alpha, y_1) = aF^{-1}_1(\alpha, y_1)$, where $F^{-1}(\alpha, y_1)$ is the inverse uncertainty distribution of $f_1(\tau_1, y_1)$. If follows from Theorem 3.6 that

$$H[\xi] = ak \int_0^1 \int_0^1 F^{-1}_1(\alpha, y_1) \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\alpha d\Psi_1(y_1) = |a|H[\xi_1].$$

If $a < 0$, then $af(\tau_1, y_1)$ has an inverse uncertainty distribution $F^{-1}(\alpha, y_1) = aF^{-1}_1(1-\alpha, y_1)$.

If follows from Theorem 3.6 that

$$H[\xi] = ak \int_0^1 \int_0^1 F^{-1}_1(\alpha, y_1) \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\alpha d\Psi_1(y_1) = |a|H[\xi_1].$$

If $a = 0$, then we immediately have $H[\xi_1] = 0 = |a|H[\xi_1]$. Thus we always have $H[\alpha \xi_1] = |a|H[\xi_1]$.

Step 2: We prove $H[\xi_1 + \xi_2] = H[\xi_1] + H[\xi_2]$. The inverse uncertainty distribution of $f_1(\tau_1, y_1) + f_2(\tau_2, y_2)$ is $F^{-1}(\alpha, y_1, y_2) = F^{-1}(\alpha, y_1) + F^{-1}(\alpha, y_2)$.

It follows from Theorem 3.6 that

$$H[\xi_1 + \xi_2] = k \int_{\mathbb{R}^2} \int_0^1 (F^{-1}_1(\alpha, y_1) + F^{-1}_2(\alpha, y_2)) \frac{2\alpha - 1}{\sqrt{\alpha(1-\alpha)}} d\alpha d\Psi_1(y_1)d\Psi_2(y_2) = H[\xi_1] + H[\xi_2].$$

Step 3: For any real numbers $a$ and $b$, it follows from Steps 1 and 2 that

$$H[\alpha \xi_1 + b \xi_2] = |a|H[\xi_1] + |b|H[\xi_2].$$

The theorem is proved. $\square$

4 Other types of Entropies for Uncertain Random Variable

**Definition 4.1** Suppose that $\eta_1, \eta_2, \cdots, \eta_m$ are independent random variables with probability distributions $\Psi_1, \Psi_2, \cdots, \Psi_m$, respectively, and $\tau_1, \tau_2, \cdots, \tau_n$ are uncertain variables. Triangular entropy of uncertain random variable $\xi = f(\eta_1, \cdots, \eta_m, \tau_1, \cdots, \tau_n)$ is defined as following

$$H[\xi] = \int_{\mathbb{R}^m} \int_{-\infty}^{+\infty} S(F(x, y_1, \cdots, y_m))dx d\Psi_1(y_1) \cdots d\Psi_m(y_m),$$
where $S(t) = 1 - |1 - 2t|$ and $F(x, y_1, \cdots, y_m)$ is the uncertainty distribution of uncertain variable $f(y_1, \cdots, y_m, \tau_1, \cdots, \tau_n)$ for any real numbers $y_1, \cdots, y_m$.

**Definition 4.2** Suppose that $\eta_1, \eta_2, \cdots, \eta_m$ are independent random variables with probability distributions $\Psi_1, \Psi_2, \cdots, \Psi_m$, respectively, and $\tau_1, \tau_2, \cdots, \tau_n$ are uncertain variables. Radical entropy of uncertain random variable $\xi = f(\eta_1, \cdots, \eta_m, \tau_1, \cdots, \tau_n)$ is defined as following

$$H[\xi] = \int_{\mathbb{R}^n} \int_{-\infty}^{+\infty} S(F(x, y_1, \cdots, y_m))dx d\Psi_1(y_1) \cdots d\Psi_m(y_m),$$

where $S(t) = \sqrt[k]{1 - t}$, $k > 0$ and $F(x, y_1, \cdots, y_m)$ is the uncertainty distribution of uncertain variable $f(y_1, \cdots, y_m, \tau_1, \cdots, \tau_n)$ for any real numbers $y_1, \cdots, y_m$.

5 Conclusions

The concepts of entropies for uncertain random variables are very important and necessary to measure the degree of uncertainty. This paper put forward the elliptic entropy of uncertain random variables. We first introduced a definition of elliptic entropy for uncertain random variables. Based on this definition, several properties were derived. In the future, we plan to investigate other types of entropies of uncertain random variable such as triangular entropy, radical entropy, and we will study their mathematical properties and possible applications.

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