

Now we consider a bilevel programming with three followers in which the leader has a decision vector  $(x_1, x_2, x_3)$  and the three followers have decision vectors  $(y_{i1}, y_{i2})$ ,  $i = 1, 2, 3$ ,

$$\left\{ \begin{array}{l} \max_{x_1, x_2, x_3} y_{11}^* y_{12}^* \sin x_1 + 2y_{21}^* y_{22}^* \sin x_2 + 3y_{31}^* y_{32}^* \sin x_3 \\ \text{subject to:} \\ x_1 + x_2 + x_3 \leq 10, x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \\ (y_{11}^*, y_{12}^*, y_{21}^*, y_{22}^*, y_{31}^*, y_{32}^*) \text{ solves the problems} \\ \left\{ \begin{array}{l} \max_{y_{11}, y_{12}} y_{11} \sin y_{12} + y_{12} \sin y_{11} \\ \text{subject to:} \\ y_{11} + y_{12} \leq x_1, y_{11} \geq 0, y_{12} \geq 0 \end{array} \right. \\ \left\{ \begin{array}{l} \max_{y_{21}, y_{22}} y_{21} \sin y_{22} + y_{22} \sin y_{21} \\ \text{subject to:} \\ y_{21} + y_{22} \leq x_2, y_{21} \geq 0, y_{22} \geq 0 \end{array} \right. \\ \left\{ \begin{array}{l} \max_{y_{31}, y_{32}} y_{31} \sin y_{32} + y_{32} \sin y_{31} \\ \text{subject to:} \\ y_{31} + y_{32} \leq x_3, y_{31} \geq 0, y_{32} \geq 0. \end{array} \right. \end{array} \right.$$

A run of GA with 1000 generations shows that the Stackelberg-Nash equilibrium is

$$\begin{aligned} (x_1^*, x_2^*, x_3^*) &= (0.000, 1.936, 8.064), \\ (y_{11}^*, y_{12}^*) &= (0.000, 0.000), \\ (y_{21}^*, y_{22}^*) &= (0.968, 0.968), \\ (y_{31}^*, y_{32}^*) &= (1.317, 6.747) \end{aligned}$$

with optimal objective values

$$\begin{aligned} y_{11}^* y_{12}^* \sin x_1^* + 2y_{21}^* y_{22}^* \sin x_2^* + 3y_{31}^* y_{32}^* \sin x_3^* &= 27.822, \\ y_{11}^* \sin y_{12}^* + y_{12}^* \sin y_{11}^* &= 0.000, \\ y_{21}^* \sin y_{22}^* + y_{22}^* \sin y_{21}^* &= 1.595, \\ y_{31}^* \sin y_{32}^* + y_{32}^* \sin y_{31}^* &= 7.120. \end{aligned}$$