Uncertain Logic

Uncertain logic was designed by Li and Liu [1] in 2009 as a generalization of logic for dealing with uncertain knowledge. A key point in uncertain logic is that the truth value of an uncertain proposition is defined as the uncertain measure that the proposition is true. One advantage of uncertain logic is the well consistency with classical logic!

Definition 1 *(Li and Liu [1]) Let X be an uncertain formula. Then the truth value of X is defined as the uncertain measure that the uncertain formula X is true, i.e.,*

$$T(X) = \mathcal{M}\{X = 1\}. \tag{1}$$

When a truth function is given, the truth value of an uncertain formula may be calculated by Chen and Ralescu's theorem.

Theorem 1 *(Chen and Ralescu [2], Truth Value Theorem) Assume that $\xi_1, \xi_2, \cdots, \xi_n$ are independent uncertain propositions with truth values $a_1, a_2, \cdots, a_n$, respectively. If X is an uncertain formula containing $\xi_1, \xi_2, \cdots, \xi_n$ with truth function f, then the truth value of X is*

$$T(X) = \begin{cases} \sup\limits_{f(x_1,x_2,\cdots,x_n)=1} \min\limits_{1\le i\le n} \nu_i(x_i), \\ \qquad \text{if } \sup\limits_{f(x_1,x_2,\cdots,x_n)=1} \min\limits_{1\le i\le n} \nu_i(x_i) < 0.5 \\ 1 - \sup\limits_{f(x_1,x_2,\cdots,x_n)=0} \min\limits_{1\le i\le n} \nu_i(x_i), \\ \qquad \text{if } \sup\limits_{f(x_1,x_2,\cdots,x_n)=1} \min\limits_{1\le i\le n} \nu_i(x_i) \ge 0.5 \end{cases} \tag{2}$$

where x_i take values either 0 or 1, and v_i are defined by

$$\nu_i(x_i) = \begin{cases} a_i, & \text{if } x_i = 1 \\ 1 - a_i, & \text{if } x_i = 0 \end{cases} \tag{3}$$

for $i = 1, 2, \cdots, n$, respectively.

Truth Value Solver

Truth Value Solver is a free software for computing the truth values of uncertain formula based on truth functions. This software may be downloaded from http://orsc.edu.cn/liu/resources.htm. Now let us perform it via some numerical examples.

Example 1: Assume that $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$ are independent uncertain propositions with truth values $0.1, 0.3, 0.5, 0.7, 0.9$, respectively. Let

$$X = (\xi_1 \wedge \xi_2) \vee (\xi_2 \wedge \xi_3) \vee (\xi_3 \wedge \xi_4) \vee (\xi_4 \wedge \xi_5). \tag{4}$$

It is clear that the truth function is

$$f(x_1, x_2, x_3, x_4, x_5) = \begin{cases} 1, & \text{if } x_1 + x_2 = 2 \\ 1, & \text{if } x_2 + x_3 = 2 \\ 1, & \text{if } x_3 + x_4 = 2 \\ 1, & \text{if } x_4 + x_5 = 2 \\ 0, & \text{otherwise.} \end{cases}$$

A run of the truth value solver shows that $T(X) = 0.7$.

Example 2: Assume that $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$ are independent uncertain propositions with truth values $0.1, 0.3, 0.5, 0.7, 0.9$, respectively. Let

$$X = \text{“only 4 propositions of } \xi_1, \xi_2, \xi_3, \xi_4, \xi_5 \text{ are true”.} \tag{5}$$

It is clear that the truth function is

$$f(x_1, x_2, x_3, x_4, x_5) = \begin{cases} 1, & \text{if } x_1 + x_2 + x_3 + x_4 + x_5 = 4 \\ 0, & \text{if } x_1 + x_2 + x_3 + x_4 + x_5 \neq 4. \end{cases}$$

A run of the truth value solver shows that $T(X) = 0.3$.

Example 3: Assume that $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$ are independent uncertain propositions with truth values $0.1, 0.3, 0.5, 0.7, 0.9$, respectively. Let

$$X = \text{“odd number of propositions of } \xi_1, \xi_2, \xi_3, \xi_4, \xi_5 \text{ are true”.} \tag{6}$$

It is clear that the truth function is

$$f(x_1, x_2, x_3, x_4, x_5) = \begin{cases} 1, & \text{if } x_1 + x_2 + x_3 + x_4 + x_5 \in \{1, 3, 5\} \\ 0, & \text{if } x_1 + x_2 + x_3 + x_4 + x_5 \in \{0, 2, 4\}. \end{cases}$$

A run of the truth value solver shows that $T(X) = 0.5$.

References

[1] Li X, and Liu B, Hybrid logic and uncertain logic, *Journal of Uncertain Systems*, Vol.3, No.2, 83-94, 2009.

[2] Chen XW, and Ralescu DA, A note on truth value in uncertain logic, http://orsc.edu.cn/online/090211.pdf.