Using Fuzzy Non-linear Regression to Identify the Compensation Level among Customer Requirements in QFD

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Abstract

The QFD method is widely used in the product preliminary design and competitive analysis of different products. The overall customer satisfaction is usually set as the objective and principle of the design planning and evaluation of products. As an MADM problem, in generally, the calculation of the objective value relies on the specification of importance weights to accomplish trade-offs among different CRs. To date, many traditional methods are developed to identify the important weights of CRs, including AHP, ANP, FAHP, FANP, NLP and so on. Most of them take no consideration of the compensation level among CRs, which is an important foundation of trade-off strategy in MADM except importance weights of attributes. The neglect of that would leads to misjudge the products to be evaluated or mislocate resource among enterprise characteristics and eventually, conduct the company make wrong decisions. In this paper, we employ the method of imprecision (MoI) in QFD to surmount this inability, and establish a fuzzy non-linear regression model to identify the compensation level amongst customer requirements. At last, an illustrative example is provided to demonstrate the application and performance of the modeling approach.

Keywords: Quality function deployment (QFD), method of imprecision (MoI), fuzzy non-linear regression, compensation level, trade-off

1 Introduction

In order to enhance competitiveness and market sharing ratio, the companies all over the world are increasing concern about the voice of target customers from the initial product design period to product feedback stage. The Quality function deployment (QFD) method, originated in the late 1960s in Japan (Akao 1990), is a useful customer-driven product development tool that uses a series of structured management processes to translate the customers’ needs into the various stages of product planning, design, engineering, and manufacturing through efficient communication. And it has been successfully applied to improve product design, decision making processes, and customer satisfaction (Cristiano, et al.2001; Lager 2005; Chan and Wu 2002) in many industries, such as software development process (Chakraborty 2007), supplier selection (Bevilacqua et al.2007; Arijit et al.2010), electronics (Jeanga et al.2009), R&D projects (Stehn and Bergstrom 2009) and so on. The manipulation of QFD data can be expressed graphically in a matrix-like configuration called the House of Quality (HOQ) (Hauser and Clausing 1988). The application of QFD is a research process of multidisciplinary and multi-method. Until now, many scholars have developed a variety of methods according to the different problems in the programming, including acquisition of the voice of customers, the identification of important weights of customer requirements (CRs) and enterprise characteristics (ECs), the calculation of the relationship matrix and correlation matrix, the designation of restriction and objective function in the model.

A number of customer requirements and enterprise characteristics are taken into account in the building process of QFD model. And it’s a classic multi-attribute decision making process. Multi-attribute decision making(MADM), as one branch of multi-criteria decision(MCDM), is a very important part of engineering design. There are many methods, both informal and formal,
that support such design decision making, such as Pugh charts, SWA/WP, the analytic hierarchy process AHP, technique for order preference by similarity to an ideal solution, efficacy coefficient method, electry-tri. Regard to QFD programming, it's a classic MADM problem and many approaches have been studied. Mohamed, et al.(2010) distinguished the researches on QFD into two main approaches: the first trend represents QFD approaches based on normalized technical importance rating; the second trend represents QFD approaches based on functional relationships that reflect the interactions between CAs and ECs. In the second trend, the HOQ approach is interpreted as a general formulation of a multi-attribute design optimization problem. The level of products' customer satisfaction is calculated based on the values of each CR and the trade-off strategy between them. The methods of AHP (Armacost et al.1994), FAHP (Kong and Bai 2003; William Ho et al.2012), ANP(Karsak et al.2003) and FANP (Kahraman et al.2006; Pal et al.) are usually employed to define customer requirements and calculate the important weights of CRs. Finally, accurate number or fuzzy number is used to indicate the important weights of CRs. Besides, fuzzy and entropy methods are also used to assess the relative importance weights of CRs (Chan and Wu 2005; Hsu and Lin 2006). Considering about the differences in backgrounds, education, domain knowledge, etc. of the invested customers, Wang (2012) proposes a nonlinear programming (NLP) approach to assessing the relative importance weights of CRs, which allows customers to express their preferences on the relative importance weights of CRs in their preferred or familiar formats. The approach does not require any transformation of preference formats and thus can avoid information loss or information distortion. These methods typically rely on a summation of weighted attributes to accomplish trade-offs among competing objectives. However, the selection of optimal solution in multi-attribute decision making (MADM) relies not only on the important weights of attributes, but also the compensation level in them. Compensation refers to a willingness to allow high performance on one attribute to compensate for low performance on another. In general, the compensation level is denoted with $s$. As another trade-off strategy except weights-sum method, compensation strategy plays an important role in the MADM. It has been early identified that weighted-sum aggregation of preferences cannot always identify all Pareto points for a design and runs the risk of missing "optimal" options without considering compensation level in attributes and default $s = 1$. The product satisfaction and prioritization from customers stem from the important weights of CRs and the willingness degree customers allow high performance on one attribute to compensate for low performance on another. But none of the above approaches applied in QFD realized the effect and influence of compensation level among CRs. The Method of Imprecision (MoI), as a formal theory for the manipulation of preliminary design information that represents preferences among design alternatives, offering different trade-off strategies for MADM problems through the foundation of aggregation functions, including non-compensating and compensating. Thus, in order to truly reflect the customer satisfaction of products and correctly conduct the product preliminary design, we attempt to develop a fuzzy QFD programming model combining the Method of Imprecision. In the model, we take the compensation level among CRs into account and use fuzzy non-linear regression method to identify it.

The rest of the paper is organized as follows. In the next section, a literature review about the application of MoI in QFD and the method to identify the compensation level $s$ in MADM is presented. In section 3, a fuzzy programming approach is put forward to address the uncertainty associated with customer requirements in QFD by using MoI and a fuzzy non-linear regression model is proposed to identify the compensation level $s$. In section 4, an illustrative example is presented to demonstrate the proposed approach and a result analysis is given. Finally, some conclusions are drawn in section 5.

2 Literature Review

The foundation of an aggregation function which represents the determination of a trade-off strategy in attributes is a crucial part of any MADM scheme. Two aggregation functions have been used for multi-attribute decision making, one which provides a compensation between the criteria, and one which does not compensate. Compensation refers to a willingness to allow high performance on one attribute to compensate for low performance on another and is a property
Table 1: Axioms of the MoI for aggregation functions

<table>
<thead>
<tr>
<th>Axiom</th>
<th>Description</th>
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<tbody>
<tr>
<td>AF.1</td>
<td>Monotonicity: If ( \mu_n \leq \mu'_n ) and ( \omega_n \leq \omega'_n ), then ( P((\mu_1, \omega_1), \ldots, (\mu_n, \omega_n)) \leq P((\mu'_1, \omega'_1), \ldots, (\mu'_n, \omega'_n)) ).</td>
</tr>
<tr>
<td>AF.2</td>
<td>Commutativity: ( P((\mu_1, \omega_1), \ldots, (\mu_j, \omega_j), \ldots, (\mu_n, \omega_n)) = P((\mu_1, \omega_1), \ldots, (\mu_n, \omega_n), \ldots, (\mu_j, \omega_j)) ) for all ( i, j ).</td>
</tr>
<tr>
<td>AF.3</td>
<td>Continuity: ( \lim_{n \to \infty} P((\mu_1, \omega_1), \ldots, (\mu_n, \omega_n)) = \mu ) for all ( \omega_1, \ldots, \omega_n \geq 0 )</td>
</tr>
<tr>
<td>AF.4</td>
<td>Idempotency: ( P((\mu_1, \omega_1), \ldots, (\mu_n, \omega_n)) = P((\mu_1, \omega_1), \ldots, (\mu_k, \omega_k), \ldots, (\mu_n, \omega_n)) ) for all ( k ).</td>
</tr>
<tr>
<td>AF.5</td>
<td>Annihilation: ( P((\mu_1, \omega_1), \ldots, (0, \omega), \ldots, (\mu_n, \omega_n)) = 0 ) for all ( \omega \neq 0 ).</td>
</tr>
<tr>
<td>AF.6</td>
<td>Self-scaling weights: ( P((\mu_1, \omega_1 t), \ldots, (\mu_n, \omega_n t)) = P((\mu_1, \omega_1), \ldots, (\mu_n, \omega_n)) ) for all ( \omega_1, \ldots, \omega_n \geq 0; \omega_1 + \ldots + \omega_n, t &gt; 0 ).</td>
</tr>
<tr>
<td>AF.7</td>
<td>Zero weights: ( P((\mu_1, \omega_1), \ldots, (\mu_k, 0), \ldots, (\mu_n, \omega_n)) = P((\mu_1, \omega_1), \ldots, (\mu_k-1, \omega_{k-1}), (\mu_k+1, \omega_{k+1}), \ldots, (\mu_n, \omega_n)) ).</td>
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of a decision rather than a design. When the overall preference for the performance of a design is limited by the attribute with the lowest performance, the decision making problem is said to be non-compensating, and the aggregation function used is the simple minimum. In this case, weights are immaterial, and are not included in the calculations. When good performance on one attribute is perceived to partially compensate for lower performance on another, the problem is called compensating, and the geometric weighted mean or product of powers has been used. The compensation strategy is the most common situation in multi-attribute decision making (MADM) decision. The MoI developed a formal description of restrictions on any aggregation function for (rational) engineering design (See Table 1). The explanation and definitions of these axioms have been presented and discussed in detail previously (Otto 1992). In particular, axiom AF.4, idempotency, and axiom AF.5, annihilation, are fundamental to MoI, distinguishing it from other decision methods. The idempotency axiom states that if several variables with identical preferences are combined, the overall preference must be the same as the (identical) preferences on the individual variables. The annihilation axiom states that if the preference for any one attribute of the design sinks to zero (unacceptable) then the overall preference for the design is zero. Usually, there’s some limitation to one or several design attributes, once the value of this attribute out of the limitation, the designation fail.

Then, a family of aggregation operators \( P_s \) that satisfy the above axioms and spans an entire range of possible operators between min and max was first proposed by Scott and Antonsson in 1998 as follows,

\[
P_s(\mu_1, \mu_2; \omega_1, \omega_2) = \left(\frac{\omega_1 \mu_1^s + \omega_2 \mu_2^s + \ldots + \omega_n \mu_n^s}{\omega_1 + \omega_2 + \ldots + \omega_n}\right)^{1/s}
\]

and \( P_s \) vary with the range of \( s \) as follows,

\[
P_{-\infty} = \lim_{s \to -\infty} P_s = \text{min}
\]

(2)

\[
P_0 = \lim_{s \to 0} P_s = (\mu_1^{\omega_1} \mu_2^{\omega_2} \ldots \mu_n^{\omega_n})^{1/(\omega_1 + \omega_2 + \ldots + \omega_n)}
\]

(3)

\[
P_1 = \lim_{s \to 1} P_s = \frac{\omega_1 \mu_1 + \omega_2 \mu_2 + \ldots + \omega_n \mu_n}{\omega_1 + \omega_2 + \ldots + \omega_n}
\]

(4)

\[
P_{+\infty} = \lim_{s \to +\infty} P_s = \text{max}
\]

(5)

Note that \( P_0 \) and \( P_1 \) are the forms of the geometric mean and the arithmetic mean respectively that we often used in MADM. \( P_{-\infty} \) indicates a non-compensating strategy. Here, \( s \) indicates
the compensation level between attributes and ranges from $-\infty$ to $+\infty$; $\omega_1, \omega_2, \ldots, \omega_n$ are the importance weights of attributes and $\omega_1, \omega_i, \ldots, \omega_n \geq 0$. The combination of parameters, $s$ and $\omega_i, i = 1, 2, \ldots, n$ determines how the resources to be distributed in the attributions to obtain the the optimal objective in constrains.

Recently, a family of aggregation functions more appropriate for engineering design is presented by them (Scott and Antonsson 2005) and the concept that both the degree of compensation and weights must be considered to capture all potentially acceptable decisions was illustrated by a simple truss design example. Kemper et al. invested the effect of different aggregation function formulations on multiattribute group decision making (Kemper and Tung-King 2006; Kemper and Michael 2007). In 2007, Scott put forward an improved AHP method to quantifying uncertainty in measurement error and preference specification and trade-offs among criteria. The concept of an aggregate objective function is also used in the context of physical programming (Messac et al. 1996; Messac and Ismail-Yahaya 2002) that can successfully be integrated into both collaborative and multidisciplinary design optimization (McAllister et al. 2005). But the compensation strategies are seldom employed in QFD method. Y. Z. Chen and E. W. T. Ngai (2008) presented a methodology combined fuzzy set theory and imprecision (MoI) method, which optimize the target values of ECs and control the distribution of the development budget by varying the value of $s$. In that paper, the determination of compensation level $s$ among ECs is on the basis of the engineering knowledge and experience of the decision makers. It’s arbitrary and ad hoc. The researchers and decision makers have growing realized the role and significance of introducing the compensation strategy into multi-attribute decision and put this idea into practice. But, regarding to the key of imprecision (MoI) method-the identification of compensation level $s$, there still no an effective method. In 2000, Scott and Antonsson applied indifference points to identify the value of $s$, but the selection of the indifference points is subjectivity and finding two designs that are of exact equivalent value to a decision maker can be a challenging and time-consuming task (Thurston 2001). And in that application, two attributes were included, so only three indifferent points were needed to identify the value of important weights and compensation level. Once the number of attributes increase, the process is hardly to carry on. Actually, just like the important weights of CRs, the compensation level $s$ among them is uncertain. In our model, to be practical and easy calculating, the value of the compensation level $s$ among CRs is expressed as a symmetric triangular fuzzy number. And for discovering it based on the customer investigation, we apply fuzzy non-linear regression model in the programming.

3 Fuzzy Programming model formulation

3.1 QFD

Since the raise of QFD in the late 1960s in Japan, it has been successfully applied in many industries to improve product design and customer satisfaction, help decision-making process, and enable the prioritisation of performance measures (Carnevalli and Miguel 2008). QFD is a technique which transforms customer requirements (CRs) or the voice of customers (VOC) into engineering characteristics using the matrix called house of quality (HOQ). This matrix contains information on what to do related to CRs, how CRs to do related to ECs, relationships between CRs and ECs and among the ECs, and benchmarking data. These information makes up the mainly five components of HOQ that are displayed in the below Figure.1 (Hauser and Clausing, 1988).

(a) The Voice of the Customer: Customers have several different aspects of requirements to one kind of product. These requirements compose the voice of the customer and directly impact on the purchasing behavior of customers. In generally, the voice of the customer is listened from the market investigation.

(b) The Voice of the Company: From the point of company, some engineering characteristics are required for the designation of the product. These are referred to as the voice of the company, design requirements, engineering characteristics, product characteristics or technical requirements. The value of these engineering characteristics directly impact on the customer satisfaction.

(c) Relationship Room: The main function of QFD is to transfer the voice of the customer to the technical requirements, so the relation between CRs and ECs is the key of QFD. In gen-
eral, this relationship is determined by professional personnel for they are the people who knows the engineering characteristics best. The relationship between them tends to be vague and the traditional method is kind of subjective, so the fuzzy linear regression is gradually introduced to identify it in recent years.

(d) Technical Correlation Roof: The technical Correlation, given in the roof matrix, are used to identify the relationships among ECs. The methods to identify it are also basically divided in to traditional method (determine by professional personnel ) and fuzzy linear regression. In the most research papers about QFD, the same means is used to define these two relationships.

(e) Technical Priorities Room: In this part, the technical priorities of engineering characteristics of the developed product, including the units, the maximums, the minimums, the values of engineering characteristics from different companies’ product and the optimal solution of engineering characteristics under some constrains.

(f) Strategic Planning Room: The analysis results will be listed in the rightmost part of the HOQ based on the research needs, usually including the degrees of satisfaction of CRs of the own company’s product and its main competitors products, the improvements of the own product under the constrains from cost and technics.

In this paper, we are aimed to use fuzzy non-linear regression to identify the compensation level S between CRs based on the data: the value of ECs, relationship between CRs and ECs, and overall customer satisfaction of the products invested from customers. So we will mainly make use of the above parts of a, b, c and e in HOQ. The technical correlation should be taken into consideration in the optimal process next when the value of s is certain.

3.2 Problem description and notation

As an effective and widely applicable method delivering the customer requirement to enterprise characteristics, the main function using QFD is to help the enterprises to proceed competition analysis and product planning. The objective and principle of that is the overall customer satisfaction of products. An important issue in the QFD programming is the foundation of objective function. The effectiveness of that directly impacts on the following analysis and decision. Due to the deficiencies of traditional weighted-sum method, we introduce the compensation level s value. In order
to certain the value of $S$ with the fuzzy non-linear regression method, the below issues need to be processed: Identification of CRs and their important weights; Calculation and normalization of relationship matrix; Normalization of the values of ECs; Investigation of prioritization and overall customer satisfaction of given products; Derivation of overall customer satisfaction.

Firstly, the notion used in the HOQ can be summarized as follows:

$$CR_i$$ is the $i$th customer requirement, $i = 1, 2, \ldots, m$;

$$EC_j$$ is the $j$th engineering characteristic, $j = 1, 2, \ldots, n$;

$$Cus_q$$ is the $q$th customer, $q = 1, 2, \ldots, g$;

$$Pro_p$$ is the $p$th product, $p = 1, 2, \ldots, k$;

$R_{ij}$ is the strength of the relation measure between $CR_i$ and $EC_j$;

$w_i$ is the relative important weight of $CR_i$, $i = 1, 2, \ldots, m$;

$x_{pj}$ is the level of attainment of $EC_j$ of product $p$, $0 \leq x_{pj} \leq 1, j = 1, 2, \ldots, n, p = 1, 2, \ldots, k$;

$y_{ji}$ is the level of attainment of $CR_j$ of product $p$, $0 \leq y_{ji} \leq 1, j = 1, 2, \ldots, m, p = 1, 2, \ldots, k$;

$yo_p$ is observation overall customer satisfaction level of product $p$, $p = 1, 2, \ldots, k$;

$yo_{pq}$ is observation level of product $p$ from customer $q$, $p = 1, 2, \ldots, k, q = 1, 2, \ldots, g$;

$se_p$ is the sequence number of attainment of product $p$, $p = 1, 2, \ldots, p$;

$seo_p$ is the observation sequence number of product $p$, $p = 1, 2, \ldots, k$;

$seo_{pq}$ is the observation sequence number of product $p$ from customer $q$, $p = 1, 2, \ldots, k, q = 1, 2, \ldots, g$;

$Y_p$ is the level of attainment of product $p$, $p = 1, 2, \ldots, k$;

$l_{pj}$ is the value of the $EC_j$ of product $p$, $j = 1, 2, \ldots, n, p = 1, 2, \ldots, k$;

$s$ is the compensation level between CRs;

$h$ is the fitting parameter in the fuzzy non-linear regression.

### 3.3 Identification of CRs and their relative importance weights

The first step to implement QFD is to identify CRs and their importance weights. A variety of approaches have been proposed in the QFD literature. AHP is a method that can be used to reconcile differences (inconsistencies) in managerial judgements and perceptions, and better resolve trade-offs. AHP treats the decision as a system so as to help decision-makers better organise their thoughts (Handfield et al. 2002, Vaidya and Kumar 2006). By reducing complex decisions to a series of pairwise comparisons, then synthesising the results, decision-makers arrive at the best decision with a clear rationale for that decision (Jing Dai and Jennifer Blackhurst 2012). In our study, we apply AHP method to identify CRs and their relative importance weights.

### 3.4 Calculation and normalization of relationship matrix

The relationship matrix is the key to link the voice of customers with the enterprise characteristics of products. There are mainly two classes of methods to identify it: determine based on experts’ engineering knowledge (Y. Z. Chen and E. W. T. Ngai 2008; Amy H. I. Lee et al. 2010) and experience or identify through regression method, which is developed into fuzzy regression method later and widely used (Karsak EE 2004; Fung RYK et al. 2006). But when fuzzy regression approaches based on LP are applied to estimate the relationships in QFD, a number of regression coefficients tend become crisp and due to the characteristics of LP (Zeynep 2010). After, nonlinear programming-based fuzzy regression, considering both the center values and the spread values of the parameter estimates in the modeling phase, is developed (Chen Y, Chen L 2006). In this paper, we employ the first method. The technical express the strength of relationships in linguistic terms, such as weak, medium or strong, which are then translated into crisp numerical or predefined fuzzy numbers. In the present paper, $r_{ij}$ is interpreted by the numerical scale 1-3-5-7-9 and normalized as follows,

$$r_{ij} = r_{ij} / \sum_{j=1}^{n} r_{ij}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.$$  (6)
3.5 Normalization of the values of ECs

In the design process of one product, some different enterprise characteristics relating to the same or different customer requirements need to be decided. But usually, the properties, units and values range of these ECs extremely diverse. In order to cover all types of inputs, the values of the ECs \( l_j \) \((j = 1, \ldots, n)\) should be normalized to a scale \([0,1]\), where 1 represents the total attainment of the given EC. ECs of a product can be classified into two types: the ” smaller-the-better type(S-type)” and ” larger-the better type(L-type)” (Chen et al. 2004) and normalized using the following transformation. Below we take product p as an example.

\[
x_{pj} = \frac{l_{pj} - l_{pj}^{min}}{l_{pj}^{max} - l_{pj}^{min}} \quad (7)
\]

\[
x_{pj} = \frac{l_{pj} - l_{pj}^{min}}{l_{pj}^{max} - l_{pj}^{min}} \quad (8)
\]

Where \( X_{pj} \) \((j = 1, \ldots, n, p = 1, 2, \ldots, k) \) is the level of attainment of \( EC_j \) \((j = 1, \ldots, n)\) of product p and the value range of \( X_{pj} \) is \([0; 1]\). (7)(8) are for S-type and L-type ECs respectively. For S-type ECs, \( l_{pj}^{max} \) is the maximum value of \( EC_j \) that matches the performance of the main competitors and \( l_{pj}^{min} \) is the minimized physical limit. Conversely, for L-type ECs, \( l_{pj}^{min} \) is the minimum value of \( EC_j \) that matches the performance of the main competitors and \( l_{pj}^{max} \) is the maximized physical limit.

3.6 Investigation of prioritization and overall customer satisfaction

One of the foundations of our fuzzy non-linear regression is the investigation of the overall customer satisfaction and prioritization of given products from customers. The investigation results will be indicated with \( yo_p \), \( seo_p \) and used in the object function and constrains of the regression model. The procedures are as follows respectively.

The procedure to certain \( seo_p \):

**Step 1:** Let all customers list the sequence number \( seo_{pq} \) of each product between 1 and k with the principle that the worse product gets the lower sequence number, \( p = 1, 2, \ldots, k; \quad q = 1, 2, \ldots, g; \)

**Step 2:** Sum up the \( seo_{pq} \) of each product;

**Step 3:** Determine the prioritization of products preliminary;

**Step 4:** If \( seo_i = seo_j, i, j = 1, 2, \ldots, k, i \neq j \), let the customers reorder them;

**Step 5:** The observation sequence number \( seo_{p}, p = 1, 2, \ldots, k \) of given products are determined ultimately.

The procedure to certain \( yo_p \) :

**Step 1:** Let all customers list the customer satisfaction \( yo_{pq} \) of each product between 1 and 9 with the principle that 1 indicates very poor and 9 indicates very good, \( p = 1, 2, \ldots, k; \quad q = 1, 2, \ldots, g; \)

**Step 2:** Sum up \( yo_{pq} \) of each product;

**Step 3:** Normalization the customer satisfaction \( yo_p \) of each products: \( yo_p = \sum_{q=1}^{g} yo_{pq} \), \( p = 1, 2, \ldots, k; \)

**Step 4:** The observation overall customer satisfaction level \( yo_{p}, p = 1, 2, \ldots, k \) of given products are determined.

3.7 Derivation of overall customer satisfaction

The combination of QFD and MOI and the calculation of compensation level \( s \) by fuzzy non-linear regression are the mainly innovation of this paper. The triangular number and trapezoidal fuzzy number are two kind of fuzzy numbers that are commonly used. Here, we set up \( s \) as a systematic triangular fuzzy number, and defined it as:

\[
\tilde{S} = (S^C, S^S). \quad (9)
\]
Where $S^C$ is the center value of $\tilde{S}$, $S^S$ is the spread value. $S^L, S^R$ are the upper and lower limit of $\tilde{S}$, $S^L = S^C - S^S$, $S^R = S^C + S^S$. Then the membership function of $\tilde{S}$ is as below:

$$u_{\tilde{S}}(S) = \begin{cases} 1 - \left| \frac{S - S^C}{S^S} \right|, & \text{if } S^L \leq S \leq S^R \\ 0, & \text{otherwise} \end{cases}$$ (10)

Then the overall customer satisfaction of product $p$ can be expressed as follows,

$$\tilde{Y}_p = \left( \frac{w_1 y^1_{p1} + w_2 y^2_{p2} + \ldots + w_m y^m_{pm}}{w_1 + w_2 + \ldots + w_m} \right)^{1/\tilde{S}}, p = 1, 2, \ldots, k.$$ (11)

If $\omega_i, i = 1, 2, \ldots, m$ is normalized to be the relative important weights of CRs, then formulation (11) can be rewritten as

$$\tilde{Y}_p = \left( \sum_{i=1}^{m} \omega_i y^i_{p1} \right)^{1/\tilde{S}}, p = 1, 2, \ldots, k.$$ (12)

And the level attainment of CR $j$ of product $p$ $y_{pi}$ can be obtained with the enterprise characteristics:

$$y_{pi} = \sum_{j=1}^{n} r_{ij} x_{pj}, i = 1, 2, \ldots, m, p = 1, 2, \ldots, k.$$ (13)

### 3.8 Development of the fuzzy non-linear regression model

Based on the fuzzy arithmetic algorithm and fuzzy extension principle, the formulation 11 can be denoted as:

$$\tilde{Y}_p = F(Y_p, \tilde{S}) = (F(Y_p, S^L), F(Y_p, S^C), F(Y_p, S^R)).$$ (14)

$F(Y_p, S^C)$ is the center value of the fuzzy output $\tilde{Y}_p$, and $F(Y_p, S^L)$ and $F(Y_p, S^R)$ are the upper and lower limits of $\tilde{Y}_p$ respectively that can be expressed as follows. But the fuzzy output obtained by the exponent operations of the symmetric triangular fuzzy number is not symmetric or triangular.

$$F(Y_p, S^R) = (\sum_{i=1}^{m} w_i y^i_{p1})^{1/S^R}. $$ (15)

$$F(Y_p, S^C) = (\sum_{i=1}^{m} w_i y^i_{p1})^{1/S^C}. $$ (16)

$$F(Y_p, S^L) = (\sum_{i=1}^{m} w_i y^i_{p1})^{1/S^L}. $$ (17)

Tanaka et al. (1988) delineated a classical fuzzy linear regression function, the aim of which is to minimize the total fuzziness of the predicted values for the dependent variables. To date, the criteria in the usage of fuzzy regression to identify target parameter usually is minimize the fuzzyness, minimize the distance between the observations and fuzzy outputs or the combination of them. The first criteria is the most common one. In this paper, we set minimization the sum of spread values of fuzzy outputs as the goal of our model and search the optimal $s$ value under the constraints as follows,

$$\min \Delta = \sum_{p=1}^{k} (F(Y_p, S^R) - F(Y_p, S^L))$$ 18(a)

Subject to

$$y_{op} \in [F(Y_p, \tilde{S})]_h, \quad p = 1, 2, \ldots, k.$$ 18(b)

$$s_{eo} = s_{ep}, p = 1, 2, \ldots, k.$$ 18(c)
Where the $\Delta$ indicates the sum of spread values of the fuzzy outputs. And according to formulation (12), the system fuzziness is defined as total covering area of fuzzy outputs as follows:

$$\Delta = \sum_{p=1}^{k} F(Y_p, S^R) - F(Y_p, S^L) = \sum_{p=1}^{k} \left( \left( \sum_{i=1}^{m} w_i y_{pi}^{S^R} \right)^{1/S^R} - \left( \sum_{i=1}^{m} w_i y_{pi}^{S^L} \right)^{1/S^L} \right) \tag{19}$$

11(b) and 11(c) are two constrains that the regression model should be subjected to. 11(b) indicates that the observed overall customer satisfaction level of product $p$ should be included in the $h$-level set of the fuzzy output $F(Y_p, \bar{S})$.

The $0 \leq h < 1$ set of $F(Y_p, \bar{S})$ is expressed as the following interval:

$$[F(Y_p, \bar{S})]_h = [F(Y_p, \bar{S})]_{hL}^L, \ [F(Y_p, \bar{S})]_{hR}^R. \tag{20}$$

$[F(Y_p, \bar{S})]_{hL}^L$ and $[F(Y_p, \bar{S})]_{hR}^R$ indicate the lower and upper limit of the $h$-level set of fuzzy output respectively. Here, unlike the traditional fuzzy linear regression that the $h$-level set of the fuzzy output is calculated with the arithmetic algorithms of triangular fuzzy numbers and represented with the lower and upper limit $F(Y_p, S^L), F(Y_p, S^R)$, we give the interval with $h$-level set of input parameter $\bar{S}$.

The lower and upper limit of $h$-level set of $s$ are calculated as:

$$S^L_h = S^L + hS^s \tag{21}$$
$$S^R_h = S^R - hS^s \tag{22}$$

The lower and upper limit of the $h$-level set of fuzzy output $F(Y_p, \bar{S})$ can be expressed as

$$[F(Y_p, \bar{S})]_{hL}^L = F(Y_p, S^L + hS^s) \tag{23}$$
$$[F(Y_p, \bar{S})]_{hR}^R = F(Y_p, S^R - hS^s). \tag{24}$$

Then the inclusion relation $y_{op} \in [F(Y_p, \bar{S})]_{hL}^L, p = 1, 2, \ldots, k$, can be rewritten as

$$F(Y_p, S^L_{hL}) \leq y_{op} \leq F(Y_p, S^R_{hR}) \tag{25}$$
$$F(Y_p, S^L + hS^s) \leq y_{op} \leq F(Y_p, S^R - hS^s). \tag{26}$$

The selection of a proper value of $h$ is important because it determines the ranges of the intervals. A higher $h$ value, yields a wider spread, the resulting predicted intervals possess a higher fuzziness (Kim et al., 1996). $h \in [0, 1)$, which is referred to as a measure of goodness of fit, is selected by the decision-maker. The criteria used to select the $h$ value are generally ad hoc, and $h$ values in earlier studies vary widely ranging from 0 to 0.9. When the data set is sufficiently large $h$ could be set to 0, whereas a higher $h$ value is suggested as the size of the data set becomes smaller (Tanaka Watada, 1988). In this paper, we execute the process of fuzzy regression with the data from 5 products and we set $h = 0.5$.

Given $l_{ij}, R_{ij}, \omega_i$, $\bar{S}$, the overall customer satisfaction of given products will change with the variation of $\bar{s}$, so does the prioritization of the products. So once the priority of given products is certain, the range of compensation level $s$ is determined as $[S_{min}, S_{max}]$. In this paper, we make both of the lower and upper limit $S^L, S^R$ of $\bar{S}$ included in the range, that is $S^L \geq s_{min}, S^R \leq s_{max}$.

Then the formulation (18) can be rewritten as

$$\min \Delta = \sum_{p=1}^{k} \left( \left( \sum_{i=1}^{m} w_i y_{pi}^{S^R} \right)^{1/S^R} - \left( \sum_{i=1}^{m} w_i y_{pi}^{S^L} \right)^{1/S^L} \right) \tag{27a}$$

Subject to

$$\left( \sum_{i=1}^{m} w_i y_{pi}^{S^L} \right)^{1/S^L} \leq y_{op} \leq \left( \sum_{i=1}^{m} w_i y_{pi}^{S^R} \right)^{1/S^R}, \ p = 1, 2, \ldots, k \tag{27b}$$
$$S^L \geq s_{min}, S^R \leq s_{max} \tag{27c}$$
$$y_{pi} = \sum_{j=1}^{n} r_{ij} x_{pj}, \ i = 1, 2, \ldots, m, p = 1, 2, \ldots, k \tag{27d}$$
$$S^L_h = S^L + hS^s, S^R_h = S^R - hS^s \tag{27e}$$
4 An illustrative example

In order to demonstrate the applicability of the proposed fuzzy non-linear regression model, the example of the development of a new type of motor car that was given by Chen et al. (2005) is used and the results are presented and discussed in this section.

4.1 Construction of the HOQ

To identify the customer requirements for motor car, a questionnaire survey was conducted to collect the voice from customers. Finally, five major CRs are identified and their relative weights are determined by AHP. Based on the design teams experience and expert knowledge on this product, five ECs that related to the five customer requirements are determined. At the same time, the relationship matrix between ECs and CRs is calculated and normalized with the method given in section 3.3. The data of five motor cares from different companies against to the related ECs are also collected for the later, the HOQ of the motor car design is constructed and shown in figure 2.

![Figure 2: The HOQ for the motor car](image)

4.2 Normalization of the values of the ECs

As indicated in figure 2, $EC_1, EC_3$ are S-type and $EC_2, EC_4, EC_5$ are L-type. Normalize them based on formulation(7) and (8) respectively. The normalization result for the five ECs of the five different motor cars are summarized in table 2.
Table 2: Normalization of the ECs of the five products

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pro1</td>
<td>0.5000</td>
<td>0.5714</td>
<td>0.1176</td>
<td>0.8571</td>
<td>0.6667</td>
</tr>
<tr>
<td>Pro2</td>
<td>0.8750</td>
<td>0.4286</td>
<td>0.5882</td>
<td>0.7857</td>
<td>0.8889</td>
</tr>
<tr>
<td>Pro3</td>
<td>0.8750</td>
<td>0.7143</td>
<td>0.9412</td>
<td>0.7143</td>
<td>0.6667</td>
</tr>
<tr>
<td>Pro4</td>
<td>0.6250</td>
<td>0.1429</td>
<td>0.7059</td>
<td>0.1429</td>
<td>0.2222</td>
</tr>
<tr>
<td>Pro5</td>
<td>0.1250</td>
<td>0.7143</td>
<td>0.8235</td>
<td>0.5000</td>
<td>0.7778</td>
</tr>
</tbody>
</table>

4.3 Investigation for customer satisfaction and sequence of the products

To execute the fuzzy non-linear regression for $\tilde{S}$, the customer satisfaction and the prioritization of the five motor cars were investigated from 20 customers. Go through the steps given in section 3, the result are summarized in table 3. We can see that, the priority of products directly invested from customer is: $Pro_3 \succ Pro_2 \succ Pro_5 \succ Pro_1 \succ Pro_4$ and it consists with the priority deduced from the overall customer satisfaction of products. And product 3 gets the highest customer satisfaction while product 4 gets the lowest.

Table 3: The customer satisfaction and sequence number of the five products invested from customers

<table>
<thead>
<tr>
<th></th>
<th>$Pro_1$</th>
<th>$Pro_2$</th>
<th>$Pro_3$</th>
<th>$Pro_4$</th>
<th>$Pro_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>seq$_p(p = 1, 2, 3, 4, 5)$</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>yo$_p(p = 1, 2, 3, 4, 5)$</td>
<td>0.542</td>
<td>0.700</td>
<td>0.805</td>
<td>0.434</td>
<td>0.578</td>
</tr>
</tbody>
</table>

4.4 Fuzzy non-linear regression

To execute the fuzzy non-linear regression, the lower and upper limits $s_{min}$, $s_{max}$, as one of the conditions in regression model, should be determined firstly.

Once the values of ECs, the relationship matrix, and the relative important weights of customer requirements are given, the overall customer satisfaction becomes a function of compensation level $s$ value and varies with the change of $s$ value. We plot the graphs of these five functions (See figure 3) and we list the overall customer satisfaction of five products at some key points (See table 4).

Table 4: overall customer satisfaction of five products with the change of $s$ value

<table>
<thead>
<tr>
<th></th>
<th>$s = -50$</th>
<th>$s = -5.3800$</th>
<th>$s = 0$</th>
<th>$s = 1$</th>
<th>$s = 7.4050$</th>
<th>$s=50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(Y_1, s)$</td>
<td>0.1220</td>
<td>0.1653</td>
<td>0.4440</td>
<td>0.5177</td>
<td>0.7014</td>
<td>0.8360</td>
</tr>
<tr>
<td>$F(Y_2, s)$</td>
<td>0.4406</td>
<td>0.5368</td>
<td>0.6592</td>
<td>0.6830</td>
<td>0.7671</td>
<td>0.8462</td>
</tr>
<tr>
<td>$F(Y_3, s)$</td>
<td>0.6989</td>
<td>0.7636</td>
<td>0.7943</td>
<td>0.8007</td>
<td>0.8389</td>
<td>0.9092</td>
</tr>
<tr>
<td>$F(Y_4, s)$</td>
<td>0.1452</td>
<td>0.1653</td>
<td>0.3044</td>
<td>0.3938</td>
<td>0.6032</td>
<td>0.6807</td>
</tr>
<tr>
<td>$F(Y_5, s)$</td>
<td>0.2710</td>
<td>0.3275</td>
<td>0.5036</td>
<td>0.5546</td>
<td>0.7014</td>
<td>0.7943</td>
</tr>
</tbody>
</table>

From the graphs and data analysis, we can get the below conclusions: 1, The functions are monotonous increasing: the bigger the $s$ value is, the higher the overall customer satisfaction. When the value of $s$ small or large enough, the overall customer satisfaction of products are getting near the minimum and maximum of their enterprise characteristics. For example, when $s = -50$, the overall customer satisfaction of product 4 is 0.1452 while the smallest value of ECs is 0.1429. They are very close. 3, The product 3 and 2 always get a greater overall customer satisfaction than other three products no matter what changes on $s$ value. The curves of product 1, 4 and 5 intersect at the points marked with asterisk with $s=5.38$ and 7.41 respectively. When the $s$ value changes in the
interval [-5.3800,7.4050], the overall customer satisfaction of five products fluctuate markedly but their priority remain unchanged. 4, The overall customer satisfaction resulted from the weighted-sum method are got with \( s = 1 \) and the points are marked with red circle. 5, With the change of \( s \) value, not only the customer satisfaction of five products changes, but also the priority of them. Particularly, the overall customer satisfaction of product 4 changed from the lowest one to the third surpassed product 1 and product 5. Based on the sequence numbers of five products in Table 3, we can deduce that \( s_{\text{min}} = -5.3800 \) and \( s_{\text{max}} = 7.4050 \).

Next, we proceed the fuzzy non-linear regression to identify the fuzzy number \( \tilde{S} \) on the basis of formulation (27). The process is as follows:

**Step 1:** Set \( s_{\text{min}} = -5.3800 \), \( s_{\text{max}} = 7.4050 \) and \( h = 0.5 \);

**Step 2:** Calculate the minimum feasible solution of the spread value \( S^S \) when the center value \( S^C \) changes from \( s_{\text{min}} \) to \( s_{\text{max}} \), then record the corresponding value of \( S^C, S^S \) and total fuzzyness \( \Delta \) of five products. The specific procedure is as follows:

For \( S^C = s_{\text{min}} : 0.001 : s_{\text{max}} \)

For \( S^S = 0.001 : 0.001 : (s_{\text{max}} - s_{\text{min}})/2 \)

If \( S_L \geq s_{\text{min}} \) and \( S_R \leq s_{\text{max}} \)

Calculate the h-level set of fuzzy output of five products;

If the observation level of the five products are included in the above h-level set

Calculate the total fuzzyness of five products;

Record the value of \( S^C, S^S \) and \( \Delta \);

Break;

End

End

End;

**Step 3:** Identify \( \tilde{S} \) that corresponding to the minimum total fuzzyness of overall customer satisfaction;

Then we get the curves (See figure 15,16) reflecting the changes of total fuzzyness of five products;
and spread value $S^S$ with the increase of the center value $\bar{S}$. From the figures, we can see that with the augment of $S^C$, the total fuzzyness $\Delta$ and the spread value $S^S$ monotone decrease firstly and then monotone increase and both of them get their minimum at the point with $S^C = 1.6250$. It’s not difficult to understand that the total fuzzyness $\Delta$, as a function of $S^R$ and $S^L$ has a close trend with the spread value $S^S$. Based on the set of condition 27(b), the finally range of the $S^C$ is limited in $[-0.5850,3.4300]$. The finally optimal solution is $\bar{S} = (1.6250,0.3700)$ with the minimum value of total fuzzyness $\Delta = 0.1386$ and $S^L = 1.2550, S^R = 1.9950$. Even the lower limit of $\bar{S}$ is greater than 1 (the compensation level in weights-sum method), an extension to this result is the overall customer satisfaction of products resulted from traditional weights-sum method may be underestimated.

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**Fig.4. Change of total fuzzyness of five products with the augment of $S^C$**

Next, we figure out the trends of membership degree and fuzzyness degree of five products with the change of $S^C$ in the feasible region (Figure. 6∼10). The trends of fuzzyness degree of products consistent with the total fuzzyness of five product while the trends of membership degree are just opposite. They monotone increase at first and then monotone decrease. Each of the products gets their maximum membership degree 0.9986, 0.9973, 0.9978, 0.9984, 0.9982 at $S^C = 1.4450, S^C = 1.8100, S^C = 1.6650, S^C = 1.4900, S^C = 1.5250$ respectively. Coincidentally, five products gets their minimum fuzzyness degree simultaneously when $S^C = 1.6250$. The points marked with red asterisks and black dots indicate the value of membership degree and fuzzyness degree respectively under the optimal solution(See Table.5). It can be seen that product 3 gets the lowest fuzzyness degree and highest membership degree.

**Fig.5. Change of spread value with the augment of $S^C$**

**Fig.6. Changes of membership and fuzzyness degree of $Pro_1$ with the augment of $S^C$**

**Fig.7. Changes of membership and fuzzyness degree of $Pro_2$ with the augment of $S^C$**
Table 5: Fuzzyness and Membership degree of five products ($\tilde{S} = (1.6250, 0.3700)$)

<table>
<thead>
<tr>
<th>Products</th>
<th>Fuzzyness Degree</th>
<th>Membership Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Pro_1$</td>
<td>0.0350</td>
<td>0.5108</td>
</tr>
<tr>
<td>$Pro_2$</td>
<td>0.0150</td>
<td>0.5054</td>
</tr>
<tr>
<td>$Pro_3$</td>
<td>0.0048</td>
<td>0.8892</td>
</tr>
<tr>
<td>$Pro_4$</td>
<td>0.0536</td>
<td>0.6324</td>
</tr>
<tr>
<td>$Pro_5$</td>
<td>0.0303</td>
<td>0.7270</td>
</tr>
</tbody>
</table>

Then, we depict the membership functions of products’ fuzzy outputs under the optimal solution using the computer simulation method (See Figure.11). Actually, since the spread value of $\tilde{S}$ is very little, the shape of membership functions are nearly triangular. The points marked with red asterisks indicate the membership degree of $y_{op}, p = 1, 2, \ldots, k$. 
In summary, we can get the below table showing the contrast of overall customer satisfaction resulted from our model with from weighted-sum method. It can be seen that the overall customer satisfactions of all products resulted from weighted-sum method are lower than the investigation value from customers, that is underestimate the customer satisfaction of products, while the investigation values of customer satisfaction get a high membership degree in fuzzy outputs of products. Using the fuzzy non-linear regression model to identify the compensation level among CRs and founding the aggregation function with the optimal $s$ value can derive a more real and effective overall customer satisfaction. Based on Table 6: contrast with the result from weighted-sum method ($s = 1$)

<table>
<thead>
<tr>
<th>Products</th>
<th>$S = (1.6250, 0.3700)$</th>
<th>weighted-sum method ($s = 1$)</th>
<th>investigation value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Pr_{01}$</td>
<td>(0.5322, 0.5508, 0.5672)</td>
<td>0.5177</td>
<td>0.542</td>
</tr>
<tr>
<td>$Pr_{02}$</td>
<td>(0.6886, 0.6963, 0.7036)</td>
<td>0.6830</td>
<td>0.700</td>
</tr>
<tr>
<td>$Pr_{03}$</td>
<td>(0.8024, 0.8047, 0.8071)</td>
<td>0.8007</td>
<td>0.805</td>
</tr>
<tr>
<td>$Pr_{04}$</td>
<td>(0.4155, 0.4441, 0.4691)</td>
<td>0.3938</td>
<td>0.434</td>
</tr>
<tr>
<td>$Pr_{05}$</td>
<td>(0.5663, 0.5822, 0.5966)</td>
<td>0.5546</td>
<td>0.578</td>
</tr>
</tbody>
</table>

that, the enterprises can make a correct competing analysis and product positioning. Furthermore, the identification of compensation level $s$ among CRs would do an important guidance for the following product decision-making.

5 Conclusion

In the current paper we employ the method of imprecision (MoI) in QFD. A fuzzy programming approach is put forward to address the uncertainty associated with the customer requirements and a fuzzy non-linear regression model is presented to identify the compensation level $s$. MoI provides a family of aggregation operators and it can identify all Pareto points for a design by
varying the value of $s$, in which the weighted sum method is only a special case with default $S = 1$. The selection and identification of compensation level $s$ would conduct some Pareto points directly eliminated. So the neglect of it may conduct the companies to make wrong decisions or wrong product design plan. In the regression model, we express the value of $s$ as a symmetric triangular fuzzy number, and make the total fuzzyness of customer satisfaction as the objective to minimise. An illustrated example is presented to show the application and performance of the proposed fuzzy non-linear regression approach. Finally, the optimal solution $\tilde{S} = (1.6250, 0.3700)$ is obtained under default constrains. QFD practitioners can use the proposed methodology to identify the compensation level $s$ amongst the CRs and properly assess the strength and weakness of given products and make corresponding product improvement strategies.

Except the CSD evaluation and competitive analysis for given products, another important function of QFD is product preliminary design. After the identification of compensation level $s$ through regression model, how to certain the ECs to maximum the customer satisfaction in technical and cost constraints and taking into account the inner relationship between ECs would be a useful area for future research. Besides, different compensation level amongst ECs also would make different combinations of ECs representing different resource distribution achieve a same customer satisfaction. So the research on that would be meaningful and significant.

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